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OF

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BY

EMERSON E. WHITE, M.A., LL.D.

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PREFACE.

THIS book presents the elements of arithmetic in a progressive three-year course. It is designed for pupils who have received some oral training in number, and have thus been prepared to use a book with advantage. The introductory oral course is fully presented in the author's *Oral Lessons in Number.*

Part One presents a progressive series of exercises in elementary processes *with small numbers.* It aims to secure accuracy and skill, and, to this end, it combines in a natural manner the inverse processes of addition and subtraction, and of multiplication and division. These progressive elementary lessons may easily be mastered by third-year pupils.

Part Two is devoted to the elementary processes *with larger integral numbers,* and the written processes are more fully developed. The special aim is to secure accuracy and rapidity in fundamental processes. Great pains have been taken to adapt the exercises and problems to the capability of fourth-year pupils. But few definitions and principles are presented, and these, when practicable, are reached inductively.

Part Three presents the more elementary processes *with fractional numbers,* common and decimal, and their more easy and practical applications. Fractions *with small terms* are used, and these are treated in both oral and written exercises in a similar manner. The use of the greatest common divisor and the least common multiple, and other like formal methods, is avoided. A full and scientific treatment of fractions is reserved for the author's second book—the *New Complete Arithmetic.*

The entire book is carefully adapted to the capability of the pupils in the several grades for whom it is designed, and the fact is kept in mind that many of these pupils will not continue in school after the fifth school year.

CINCINNATI, O., *May* 14, 1890.

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FIRST BOOK OF ARITHMETIC.

PART ONE.

NUMBERS ONE TO TEN.

1. How many words has Helen written on her slate? If she erase 2 words, how many words will be left?

2. Helen has written 5 words on her slate: if she write 3 more, how many words will then be on her slate?

3. How many words are 4 words and 4 words? 4 words and 5 words? 4 words and 6 words?

4. Ward has 9 marbles; if he give his brother 4 marbles, how many will he have left?

5. How many marbles are 5 marbles and 4 marbles? 5 marbles and 5 marbles? 5 marbles and 3 marbles?

6. How many pencils are 6 pencils and 3 pencils? 6 pencils and 4 pencils? 6 pencils and 2 pencils?

7. How many pencils are 8 pencils less 4 pencils? 9 pencils less 4 pencils? 10 pencils less 4 pencils?

How many are:—

1 and 2?	2 and 1?	3 less 1?	3 less 2?
1 and 3?	3 and 1?	4 less 1?	4 less 3?
2 and 2?	———	4 less 2?	———
1 and 4?	4 and 1?	5 less 1?	5 less 4?
2 and 3?	3 and 2?	5 less 2?	5 less 3?
1 and 5?	5 and 1?	6 less 1?	6 less 5?
2 and 4?	4 and 2?	6 less 2?	6 less 4?
3 and 3?	———	6 less 3?	———
1 and 6?	6 and 1?	7 less 1?	7 less 6?
2 and 5?	5 and 2?	7 less 2?	7 less 5?
3 and 4?	4 and 3?	7 less 3?	7 less 4?
1 and 7?	7 and 1?	8 less 1?	8 less 7?
2 and 6?	6 and 2?	8 less 2?	8 less 6?
3 and 5?	5 and 3?	8 less 3?	8 less 5?
4 and 4?	———	8 less 4?	———
1 and 8?	8 and 1?	9 less 1?	9 less 8?
2 and 7?	7 and 2?	9 less 2?	9 less 7?
3 and 6?	6 and 3?	9 less 3?	9 less 6?
4 and 5?	5 and 4?	9 less 4?	9 less 5?
1 and 9?	9 and 1?	10 less 1?	10 less 9?
2 and 8?	8 and 2?	10 less 2?	10 less 8?
3 and 7?	7 and 3?	10 less 3?	10 less 7?
4 and 6?	6 and 4?	10 less 4?	10 less 6?
5 and 5?	———	10 less 5?	———

TO TEACHERS.—The above exercises may be recited from left to right, without questions; thus,—

1 and 3 are 4; 3 and 1 are 4; 4 less 1 is 3; 4 less 3 is 1.

8. How many 2's in 4? 2's in 6? 2's in 8? 2's in 10?

9. How many 3's in 6? 3's in 9? 4's in 8? 5's in 10?

10. What two numbers added make 2? 3? 4? 5? 6? 7? 8? 9? 10?

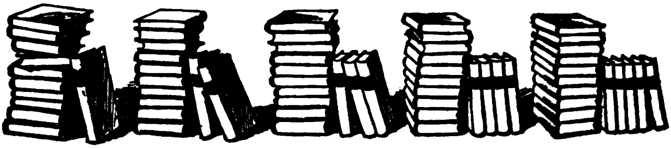
11. Into what two numbers can 2 be separated? 3? 4? 5? 6? 7? 8? 9? 10?

12. Write these figures on your slate:—

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

NUMBERS TEN TO TWENTY.

1. How many books are 10 books and 1 book? 10 books and 2 books? 10 books and 3 books? 10 books and 4 books? 10 books and 5 books?



2. How many are 10 and 1? 10 and 2? 10 and 3? 10 and 4? 10 and 5?

3. How many are 10 and 3? 13 less 3? 10 and 4? 14 less 4? 10 and 5? 15 less 5?

4. Write on your slate 11, 12, 13, 14, 15.

5. How many books are 10 books and 6 books? 10 books and 7 books? 10 books and 8 books? 10 books and 9 books? 10 books and 10 books?

6. How many are 10 and 6? 10 and 7? 10 and 8? 10 and 9? 10 and 10?

7. How many are 10 and 7? 17 less 7? 10 and 8? 18 less 8? 10 and 9? 19 less 9? 20 less 10?

8. How many must be added to 10 to make 15? 10 to make 13? 16? 17? 18? 19? 20?

9. John gave 10 cents for a slate and 8 cents for a primer; how many cents did he give for both?

10. Harry earned 16 cents and gave 6 cents for pencils; how many cents had he left?

11. Write on your slate 16, 17, 18, 19, 20.

12. Copy on your slate and read: 11, 13, 15, 17, 19, 12, 14, 16, 18, 20.

NUMBERS TWENTY TO ONE HUNDRED.

1. Grace has a numeral frame, and there are 10 balls on each wire. How many wires has the frame?

2. How many balls on 2 wires? On 3 wires? 4 wires? 5 wires? 6 wires? 7 wires? 8 wires? 9 wires? 10 wires?

3. Count from 10 to 50 by tens; thus, ten, twenty, etc.

4. Count from 50 to 100 by tens.

5. How many tens in twenty? In thirty? Forty? Fifty? Sixty? Seventy? Eighty? Ninety? One hundred?

6. How many ones in ten? In 2 tens? 3 tens? 4 tens? 5 tens? 6 tens? 7 tens? 8 tens? 9 tens? 10 tens?

7. How many are 20 and 1? 20 and 2? 20 and 3? 20 and 4? 20 and 5? 20 and 6? 20 and 7? 20 and 8? 20 and 9? 20 and 10?

8. Copy and read the following numbers: 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

9. Count by ones from 20 to 30; 30 to 40; 40 to 50; 50 to 60; 60 to 70; 70 to 80; 80 to 90; 90 to 100.

ADDITION AND SUBTRACTION.

ADDITIVE NUMBERS, ONE AND TWO.

ORAL EXERCISES.

1. THERE are 6 birds on a limb: if 2 birds fly away, how many birds will be left? 2 birds from 6 birds leave how many?

2. How many birds are 5 birds and 2 birds? 8 birds and 2 birds? 7 birds and 2 birds? 9 birds and 2 birds? 10 birds and 2 birds?

3. How many pears are 8 pears less 2 pears? 7 pears less 2 pears? 10 pears less 2 pears? 11 pears less 2 pears? 12 pears less 2 pears?

TO TEACHERS.—Children should not be taught the analysis of these problems. They should add or subtract, and give the result.

How many are:—		*How many are:—*	
1 and 1?	2 less 1?	1 and 2?	3 less 2?
2 and 1?	3 less 1?	2 and 2?	4 less 2?
3 and 1?	4 less 1?	3 and 2?	5 less 2?
4 and 1?	5 less 1?	4 and 2?	6 less 2?
5 and 1?	6 less 1?	5 and 2?	7 less 2?
6 and 1?	7 less 1?	6 and 2?	8 less 2?
7 and 1?	8 less 1?	7 and 2?	9 less 2?
8 and 1?	9 less 1?	8 and 2?	10 less 2?
9 and 1?	10 less 1?	9 and 2?	11 less 2?
10 and 1?	11 less 1?	10 and 2?	12 less 2?

NOTE.—These tables may be recited from left to right; thus, 1 and 1 are 2; 2 less 1 is 1; 2 and 1 are 3; 3 less 1 is 2.

4. How many balls are 7 balls and 2 balls? 17 balls and 2 balls? 27 balls and 2 balls? 37 balls and 2 balls?

5. How many are 6 and 2? 16 and 2? 26 and 2? 46 and 2? 66 and 2? 86 and 2? 96 and 2?

6. How many are 7 and 2? 9 less 2? 27 and 2? 29 less 2? 37 and 2? 39 less 2?

7. How many is 8 less 2? 18 less 2? 28 less 2? 48 less 2? 68 less 2? 88 less 2? 98 less 2?

8. How many are 8 and 2? 10 less 2? 18 and 2? 20 less 2? 28 and 2? 30 less 2?

9. How many are 9 and 2? 11 less 2? 29 and 2? 31 less 2? 49 and 2? 51 less 2?

WRITTEN EXERCISES.

	2	2	2	1	2	2
1. Add	7	9	8	9	6	5
2. From	9	11	10	10	8	7
Take	2	2	2	1	2	2

Copy and add:—

3.	**4.**	**5.**	**6.**	**7.**
1	2	2	2	2
1	2	2	2	1
1	2	2	1	2
1	2	2	2	1
1	2	2	1	2
1	2	2	1	1
1	2	2	2	2
1	2	2	2	1
1	2	2	2	2
1	2	2	1	1
1	2	2	2	2
1	2	1	1	1

Copy and read:—

8.	**9.**	**10.**	**11.**
10	12	37	80
13	15	39	83
14	18	44	87
19	21	46	86
17	24	52	90
16	22	57	93
20	27	63	96
25	31	66	97
23	30	69	85
26	33	70	54
29	34	74	68
28	36	78	42

NEW ADDITIVE NUMBER, THREE.

ORAL EXERCISES.

1. A boy has 5 balls in his hat and 3 in his hands: how many balls has he?

2. A boy bought 12 oranges, and gave 3 of them to his sister: how many oranges had he left?

3. How many plums are 6 plums and 3 plums? 7 plums and 3 plums?

4. How many plums are 9 plums less 3 plums? 10 plums less 3 plums? 11 plums less 3 plums?

How many are:—		*Read and complete:—*	
1 and 3?	4 less 3?	$1+3=$	$4-3=$
2 and 3?	5 less 3?	$2+3=$	$5-3=$
3 and 3?	6 less 3?	$3+3=$	$6-3=$
4 and 3?	7 less 3?	$4+3=$	$7-3=$
5 and 3?	8 less 3?	$5+3=$	$8-3=$
6 and 3?	9 less 3?	$6+3=$	$9-3=$
7 and 3?	10 less 3?	$7+3=$	$10-3=$
8 and 3?	11 less 3?	$8+3=$	$11-3=$
9 and 3?	12 less 3?	$9+3=$	$12-3=$
10 and 3?	13 less 3?	$10+3=$	$13-3=$

NOTE.—The teacher should make these tables on the board, and, after teaching the signs +, —, and =, drill the pupils until they can read and complete with rapidity.

5. How many are 6 and 3? 9 less 3? 10 less 3? 13 less 3? 12 less 3? 11 less 3?

6. How many are 7 and 3? 10 less 3? 9 and 3? 12 less 3? 8 and 3? 11 less 3? 10 and 3? 13 less 3? 11 and 3? 14 less 3?

7. How many are 7 and 3? 17 and 3? 27 and 3? 47 and 3? 57 and 3? 67 and 3? 87 and 3?

8. How many is 10 less 3? 30 less 3? 40 less 3? 60 less 3? 50 less 3? 80 less 3? 90 less 3?

To Teachers.—If a pupil hesitates in adding 3 to 18, or to any other number ending in 8, have him add 8 and 3—*the digital numbers.* Great pains should be taken to prevent counting. In subtracting 3 from 21, or from any other number ending in 1, the pupil, should, if necessary, take 3 from 11 and then 3 from 21.

9. How many are 8 and 3? 11 less 3? 18 and 3? 21 less 3? 28 and 3? 31 less 3? 48 and 3? 51 less 3? 68 and 3? 71 less 3?

10. How many are 9 and 3? 12 less 3? 29 and 3? 32 less 3? 49 and 3? 52 less 3?

11. How many 3's in 12? 3's in 6? 3's in 9?

WRITTEN EXERCISES.

	3	3	13	13	13
1. Add	8	9	12	13	15
2. From	11	12	15	16	18
Take	3	3	3	3	3
	13	23	33	23	33
3. Add	18	28	48	37	57

Copy and add:—

4.	**5.**	**6.**	**7.**	**8.**
3	3	3	3	3
3	3	3	1	3
3	3	3	1	2
3	3	3	3	2
3	3	3	1	3
3	3	3	3	2
3	3	3	1	3
3	3	3	3	3
3	3	3	1	2
3	3	3	1	2
3	1	2	1	3

Copy and read:—

9.	**10.**	**11.**
100	110	140
101	111	150
102	112	152
103	113	154
104	114	160
105	115	166
106	116	168
107	117	170
108	118	173
109	119	175
110	120	177

NEW ADDITIVE NUMBER, FOUR.

ORAL EXERCISES.

1. A farmer has 7 hogs in one pen and 4 hogs in another pen: how many hogs are in both pens?

2. A farmer has 13 hogs, and sells 4 of them: how many hogs has he left?

3. Harry has 8 marbles in one hand and 4 marbles in the other hand: how many marbles has he in both hands?

4. There are 14 pupils in a class: if 4 of them leave the class, how many pupils will remain?

5. How many lines are 7 lines and 4 lines? 9 lines and 4 lines? 10 lines and 4 lines? 8 lines and 4 lines? 9 lines and 4 lines?

6. How many lines are 11 lines less 4 lines? 12 lines less 4 lines? 10 lines less 4 lines? 13 lines less 4 lines? 14 lines less 4 lines?

How many are:—

1 and 4?
2 and 4?
3 and 4?
4 and 4?
5 and 4?
6 and 4?
7 and 4?
8 and 4?
9 and 4?
10 and 4?

5 less 4?
6 less 4?
7 less 4?
8 less 4?
9 less 4?
10 less 4?
11 less 4?
12 less 4?
13 less 4?
14 less 4?

Read and complete:—

$1 + 4 =$
$2 + 4 =$
$3 + 4 =$
$4 + 4 =$
$5 + 4 =$
$6 + 4 =$
$7 + 4 =$
$8 + 4 =$
$9 + 4 =$
$10 + 4 =$

$5 - 4 =$
$6 - 4 =$
$7 - 4 =$
$8 - 4 =$
$9 - 4 =$
$10 - 4 =$
$11 - 4 =$
$12 - 4 =$
$13 - 4 =$
$14 - 4 =$

7. How many are 7 and 4? 11 less 4? 9 and 4? 13 less 4? 8 and 4? 12 less 4? 10 and 4? 14 less 4?

8. How many are 7 and 4? 17 and 4? 37 and 4? 57 and 4? 27 and 4? 47 and 4? 67 and 4?

9. How many are 9 and 4? 19 and 4? 39 and 4? 29 and 4? 49 and 4? 59 and 4? 69 and 4?

10. How many are 8 and 4? 18 and 4? 48 and 4? 38 and 4? 28 and 4? 58 and 4? 68 and 4?

11. How many is 12 less 4? 22 less 4? 32 less 4? 52 less 4? 42 less 4? 62 less 4? 72 less 4?

12. How many is 13 less 4? 33 less 4? 43 less 4? 63 less 4? 73 less 4? 53 less 4? 83 less 4?

13. How many 4's in 8? 4's in 12? 4's in 13?

WRITTEN EXERCISES.

	14	4	14	14	14
1. Add	13	9	11	12	15
2. From	17	13	15	16	19
Take	14	4	4	4	14
	24	14	14	14	34
3. Add	27	37	39	56	26
4. From	46	56	58	75	66
Take	14	14	14	14	34

Copy and add:—

5.	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
4	4	4	4	3	4	4	1	2	4
4	4	4	4	4	3	3	2	2	4
4	4	4	2	3	2	2	4	2	4
4	4	4	1	3	4	1	3	3	3
4	4	4	3	4	1	0	2	3	3
4	4	4	4	3	4	3	4	3	4
4	4	4	2	4	4	4	4	1	4
4	4	4	3	4	4	2	0	0	2
4	4	4	4	4	3	3	3	4	0
4	4	4	1	3	3	2	2	4	3
4	3	2	4	3	4	4	0	4	4

NEW ADDITIVE NUMBER, FIVE.

ORAL EXERCISES.

1. A boy has 4 nuts in one hand and 5 nuts in the other: how many nuts in both his hands?

2. Harry had 12 nuts, and gave 5 of them to Mary: how many nuts had he left?

3. There are 9 cows in one pasture and 5 cows in another: how many cows in both pastures?

4. How many are 9 cows less 5 cows? 12 cows less 5 cows? 10 cows less 5 cows?

5. How many cents are 8 cents and 5 cents? 18 cents and 5 cents? 28 cents and 5 cents?

6. How many are 6 and 5? 4 and 5? 7 and 5? 8 and 5? 9 and 5? 10 and 5?

How many are:—

1 and 5? 6 less 5?
2 and 5? 7 less 5?
3 and 5? 8 less 5?
4 and 5? 9 less 5?
5 and 5? 10 less 5?
6 and 5? 11 less 5?
7 and 5? 12 less 5?
8 and 5? 13 less 5?
9 and 5? 14 less 5?
10 and 5? 15 less 5?

Read and complete:—

$1 + 5 =$ $6 - 5 =$
$2 + 5 =$ $7 - 5 =$
$3 + 5 =$ $8 - 5 =$
$4 + 5 =$ $9 - 5 =$
$5 + 5 =$ $10 - 5 =$
$6 + 5 =$ $11 - 5 =$
$7 + 5 =$ $12 - 5 =$
$8 + 5 =$ $13 - 5 =$
$9 + 5 =$ $14 - 5 =$
$10 + 5 =$ $15 - 5 =$

7. How many is 6 less 5? 8 less 5? 10 less 5? 12 less 5? 14 less 5?

8. How many is 7 less 5? 9 less 5? 11 less 5? 13 less 5? 15 less 5?

9. How many are 8 and 5? 13 less 5? 10 and 5? 15 less 5? 7 and 5? 12 less 5? 9 and 5? 14 less 5?

10. How many are 7 and 5? 27 and 5? 47 and 5? 67 and 5? 87 and 5? 77 and 5? 97 and 5?

11. How many are 9 and 5? 19 and 5? 39 and 5? 29 and 5? 49 and 5? 69 and 5? 59 and 5?

12. How many is 12 less 5? 22 less 5? 32 less 5? 42 less 5? 52 less 5? 62 less 5? 72 less 5?

13. How many is 13 less 5? 33 less 5? 53 less 5? 63 less 5? 83 less 5? 73 less 5?

14. How many is 14 less 5? 24 less 5? 34 less 5? 54 less 5? 44 less 5? 64 less 5?

15. How many 5's in 10? 5's in 15? 5's in 12?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
	15	15	25	25	35	35	45	45
Add	16	27	28	39	29	36	26	25

	9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
From	15	14	24	22	32	41	61	53
Take	5	5	15	15	15	25	35	25

Copy and add:—

17.	**18.**	**19.**	**20.**	**21.**
5	5	5	5	2
5	5	5	5	2
5	5	5	5	5
5	5	5	3	5
5	5	5	5	5
5	5	5	1	3
5	5	5	4	5
5	5	5	5	2
5	5	5	5	4
5	5	5	1	5
5	1	3	5	2

Copy and read:—

22.	**23.**	**24.**
200	122	244
202	217	246
212	118	248
203	216	245
213	215	247
205	223	249
215	227	250
207	234	260
217	239	280
208	235	270
218	237	290

NEW ADDITIVE NUMBER, SIX.

ORAL EXERCISES.

1. Frank caught 8 trout, and James 6 trout: how many trout did both catch?

2. A boy caught 13 trout, and gave 6 of them to a poor boy: how many trout had he left?

3. Charles gave 9 cents for pencils and 6 cents for paper: how many cents did he spend?

4. A boy wrote 15 words on his slate, and then erased 6 of them: how many words were left on his slate?

5. How many are 4 and 6? 6 and 6? 5 and 6? 7 and 6? 9 and 6? 8 and 6? 10 and 6?

6. How many is 8 less 6? 10 less 6? 9 less 6? 11 less 6? 12 less 6? 13 less 6? 15 less 6?

How many are:—

1 and 6?
2 and 6?
3 and 6?
4 and 6?
5 and 6?
6 and 6?
7 and 6?
8 and 6?
9 and 6?
10 and 6?

7 less 6?
8 less 6?
9 less 6?
10 less 6?
11 less 6?
12 less 6?
13 less 6?
14 less 6?
15 less 6?
16 less 6?

Read and complete:—

$1 + 6 =$
$2 + 6 =$
$3 + 6 =$
$4 + 6 =$
$5 + 6 =$
$6 + 6 =$
$7 + 6 =$
$8 + 6 =$
$9 + 6 =$
$10 + 6 =$

$7 - 6 =$
$8 - 6 =$
$9 - 6 =$
$10 - 6 =$
$11 - 6 =$
$12 - 6 =$
$13 - 6 =$
$14 - 6 =$
$15 - 6 =$
$16 - 6 =$

7. How many are 12 cents less 6 cents? 32 cents less 6 cents? 52 cents less 6 cents? 72 cents less 6 cents? 92 cents less 6 cents?

8. How many are 7 and 6? 27 and 6? 37 and 6? 57 and 6? 67 and 6? 87 and 6? 77 and 6?

9. How many are 8 and 6? 18 and 6? 28 and 6? 48 and 6? 58 and 6? 78 and 6? 88 and 6?

10. How many are 9 and 6? 29 and 6? 39 and 6? 59 and 6? 79 and 6? 69 and 6? 89 and 6?

11. How many is 9 less 6? 11 less 6? 13 less 6? 14 less 6? 10 less 6? 15 less 6? 16 less 6?

12. How many 6's in 12? 6's in 13? 6's in 18?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	16	46	24	56	63	66
	26	23	44	46	64	55
	22	43	36	36	46	44
	23	16	35	61	36	33
Add	36	36	46	66	25	22

	7.	**8.**	**9.**	**10.**	**11.**	**12.**
From	13	48	69	88	99	43
Take	6	26	36	46	66	16

Copy and add:—

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**
6	6	6	3	2	4	6	5	4	6
6	6	2	6	3	6	4	6	3	5
6	6	1	3	6	4	6	5	4	4
6	6	6	6	2	4	1	6	2	4
6	6	6	6	3	4	3	6	1	4
6	6	6	6	6	4	4	5	2	5
6	6	6	6	2	6	6	5	3	5
6	6	6	6	3	6	6	6	4	5
6	6	6	6	1	6	4	6	5	6
6	6	6	6	2	6	3	6	5	5
6	6	6	6	3	6	2	6	4	6
6	1	2	3	6	4	1	5	3	4

NEW ADDITIVE NUMBER, SEVEN.

ORAL EXERCISES.

1. Harry found 8 eggs in one nest and 7 eggs in another: how many eggs did he find?

2. Charles picked 9 quarts of cherries from one tree and 7 quarts from another tree: how many quarts of cherries did he pick?

3. Ethel picked 12 nice peaches, and gave her mother 7 of them: how many peaches had Ethel left?

4. A man, having 13 sheep, sold 7 of them: how many sheep had he left?

How many are:—		*Read and complete:—*	
1 and 7?	8 less 7?	$1+7=$	$8-7=$
2 and 7?	9 less 7?	$2+7=$	$9-7=$
3 and 7?	10 less 7?	$3+7=$	$10-7=$
4 and 7?	11 less 7?	$4+7=$	$11-7=$
5 and 7?	12 less 7?	$5+7=$	$12-7=$
6 and 7?	13 less 7?	$6+7=$	$13-7=$
7 and 7?	14 less 7?	$7+7=$	$14-7=$
8 and 7?	15 less 7?	$8+7=$	$15-7=$
9 and 7?	16 less 7?	$9+7=$	$16-7=$
10 and 7?	17 less 7?	$10+7=$	$17-7=$

5. How many are 2 and 7? 4 and 7? 6 and 7? 8 and 7? 10 and 7? 3 and 7? 0 and 7?

6. How many are 5 and 7? 7 and 7? 9 and 7? 6 and 7? 10 and 7? 8 and 7?

7. How many is 8 less 7? 9 less 7? 11 less 7? 13 less 7? 14 less 7? 15 less 7? 17 less 7? 16 less 7?

8. How many are 7 and 7? 17 and 7? 27 and 7? 47 and 7? 67 and 7? 87 and 7?

9. How many are 8 and 7? 28 and 7? 38 and 7? 48 and 7? 68 and 7? 88 and 7? 78 and 7?

10. How many are 9 and 7? 19 and 7? 29 and 7? 39 and 7? 59 and 7? 69 and 7? 89 and 7?

11. How many is 11 less 7? 21 less 7? 31 less 7? 41 less 7? 81 less 7? 61 less 7? 51 less 7?

12. How many is 13 less 4? 13 less 5? 13 less 6? 13 less 7? 14 less 6? 14 less 5? 14 less 7?

13. How many 7's in 14? 7's in 15? 7's in 10?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	12	17	34	21	37	27	50
	13	27	24	37	44	55	17
	14	37	64	60	77	37	68
	15	47	54	42	22	43	57
Add	16	68	74	67	33	65	77

	8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
From	21	31	45	66	83	60	50
Take	17	17	27	37	57	37	27

Copy and add:—

15.	**16.**	**17.**	**18.**	**19.**
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	7	7	7	7
7	1	2	3	4

Copy and read:—

20.	**21.**	**22.**
$15	$50	$300
$12	$84	$400
$18	$65	$500
$20	$16	$600
$25	$45	$612
$30	$70	$625
$35	$83	$666
$40	$22	$700
$42	$35	$750
$48	$24	$800
$45	$56	$880

NOTE.—The character $ denotes dollars, and is called the *dollar sign.* $210 is read 210 *dollars;* $446 is read 446 *dollars.*

NEW ADDITIVE NUMBER, EIGHT.

ORAL EXERCISES.

1. Mary has written 7 words in one line and 8 words in another line: how many words in both lines?

2. A farmer put 6 sacks of flour in his wagon, and the miller put in 8 sacks more: how many sacks of flour in the wagon?

3. A wagon contains 14 bags of wheat: if 8 bags be taken out, how many bags will be left?

4. Charles earned $16, and paid $8 for a coat: how many dollars had he left?

5. Kate has 9 plums in her right hand and 8 plums in her left hand: how many plums in both her hands?

6. How many are 2 cents and 8 cents? 3 cents and 8 cents? 4 cents and 8 cents? 6 cents and 8 cents?

How many are:—

1 and 8?
2 and 8?
3 and 8?
4 and 8?
5 and 8?
6 and 8?
7 and 8?
8 and 8?
9 and 8?
10 and 8?

9 less 8?
10 less 8?
11 less 8?
12 less 8?
13 less 8?
14 less 8?
15 less 8?
16 less 8?
17 less 8?
18 less 8?

Read and complete:—

$1 + 8 =$
$2 + 8 =$
$3 + 8 =$
$4 + 8 =$
$5 + 8 =$
$6 + 8 =$
$7 + 8 =$
$8 + 8 =$
$9 + 8 =$
$10 + 8 =$

$9 - 8 =$
$10 - 8 =$
$11 - 8 =$
$12 - 8 =$
$13 - 8 =$
$14 - 8 =$
$15 - 8 =$
$16 - 8 =$
$17 - 8 =$
$18 - 8 =$

7. How many are 3 and 8? 5 and 8? 7 and 8? 9 and 8? 10 and 8? 6 and 8? 8 and 8?

8. How many is 10 less 8? 12 less 8? 14 less 8? 16 less 8? 18 less 8? 13 less 8? 15 less 8? 17 less 8?

9. How many pecks are 4 pecks and 8 pecks? 24 pecks and 8 pecks? 44 pecks and 8 pecks?

10. How many sacks are 5 sacks and 8 sacks? 25 sacks and 8 sacks? 55 sacks and 8 sacks? 75 sacks and 8 sacks? 85 sacks and 8 sacks?

11. How many are 7 and 8? 27 and 8? 47 and 8? 67 and 8? 87 and 8? 77 and 8? 97 and 8?

12. How many are 8 and 8? 28 and 8? 38 and 8? 58 and 8? 48 and 8? 68 and 8? 78 and 8?

13. How many are 9 and 8? 19 and 8? 29 and 8? 59 and 8? 69 and 8? 89 and 8? 99 and 8?

14. How many is 12 less 8? 14 less 8? 16 less 8? 26 less 8? 36 less 8? 15 less 8? 25 less 8?

15. Begin with 4 and add to 100 by 8's; thus, 4, 12, 20, 28, 36, etc.

16. Begin with 3 and add to 67 by 8's.

17. How many 8's in 16? 8's in 17? 8's in 18?

WRITTEN EXERCISES.

Copy and add:—

1.	**2.**	**3.**	**4.**	**5.**	**6.**
8	8	8	5	6	7
8	8	4	8	7	8
8	8	8	5	4	6
8	8	2	6	5	8
8	8	8	8	7	3
8	8	8	8	6	8
8	8	8	8	8	4
8	8	8	8	8	8
8	8	8	8	8	5
8	8	8	8	8	8
8	8	8	8	8	8
8	1	2	3	4	5

Copy and read:—

7.	**8.**	**9.**
$ 7	$5.25	$10.25
$ 9	$4.12	$15.30
$15	$1.60	$25.04
$21	$5.00	$35.06
$33	$2.45	$40.50
$55	$3.06	$20.85
$66	$9.12	$ 9.63
$75	$6.44	$42.10
$82	$3.90	$50.05
$90	$2.20	$65.50
$80	$1.06	$14.10
$95	$4.50	$44.44

Note.—$5.25 is read 5 *dollars* 25 *cents*, the figures at the right of the period denoting cents, and those at the left denoting dollars.

	10.	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
	28	32	60	33	55	70	$4.75
	38	46	78	43	60	32	$5.80
	48	38	81	53	70	68	$6.35
	58	28	38	63	66	47	$7.70
	68	16	46	78	77	88	$8.85
Add	78	50	88	88	88	55	$3.44

	17.	**18.**	**19.**	**20.**	**21.**	**22.**
From	55	75	$45	$65	$4.65	$6.60
Take	38	58	$18	$28	$2.81	$4.85

23. Add 37, 28, 49, 50, 61, 37, 82, 43, and 54.

24. Add 63, 38, 45, 54, 73, 47, 88, 66, and 78.

25. Add $61, $62, $63, $64, $65, $66, $67, and $68.

NEW ADDITIVE NUMBER, NINE.

ORAL EXERCISES.

1. A school-house has 5 windows in the ends and 9 windows in the sides: how many windows in the house?

2. Mary solved 7 problems on her slate and 9 other problems on paper: how many problems did she solve?

3. Albert gave 8 cents for a slate and 9 cents for paper: how many cents did he pay for both?

4. A farmer gave 6 dollars for a sheep, and 9 dollars for a calf: how many dollars did he give for both?

5. Harry picked 13 peaches, and gave 9 of them to his playmates: how many had he left?

6. How many are 1 and 9? 2 and 9? 3 and 9? 4 and 9? 5 and 9? 6 and 9? 7 and 9? 8 and 9? 9 and 9? 10 and 9?

How many are:—		*Read and complete:—*	
1 and 9?	10 less 9?	$1 + 9 =$	$10 - 9 =$
2 and 9?	11 less 9?	$2 + 9 =$	$11 - 9 =$
3 and 9?	12 less 9?	$3 + 9 =$	$12 - 9 =$
4 and 9?	13 less 9?	$4 + 9 =$	$13 - 9 =$
5 and 9?	14 less 9?	$5 + 9 =$	$14 - 9 =$
6 and 9?	15 less 9?	$6 + 9 =$	$15 - 9 =$
7 and 9?	16 less 9?	$7 + 9 =$	$16 - 9 =$
8 and 9?	17 less 9?	$8 + 9 =$	$17 - 9 =$
9 and 9?	18 less 9?	$9 + 9 =$	$18 - 9 =$
10 and 9?	19 less 9?	$10 + 9 =$	$19 - 9 =$

7. How many is 10 less 9? 12 less 9? 14 less 9? 16 less 9? 18 less 9? 20 less 9?

8. How many is 11 less 9? 13 less 9? 15 less 9? 17 less 9? 19 less 9? 18 less 9? 16 less 9?

9. How many are 7 and 9? 27 and 9? 37 and 9? 47 and 9? 67 and 9?

10. How many are 18 and 9? 28 and 9? 38 and 9? 58 and 9? 68 and 9?

11. How many are 6 and 9? 26 and 9? 46 and 9? 66 and 9? 86 and 9? 76 and 9? 56 and 9?

12. How many are 15 and 9? 25 and 9? 35 and 9? 85 and 9? 75 and 9? 65 and 9? 55 and 9?

13. How many is 15 less 9? 25 less 9? 35 less 9? 45 less 9? 55 less 9? 65 less 9? 75 less 9?

14. How many is 12 less 9? 22 less 9? 42 less 9? 62 less 9? 52 less 9? 72 less 9? 82 less 9?

15. How many 9's in 27? 9's in 21? 9's in 39? 9's in 37? 9's in 46? 9's in 56? 9's in 60?

16. Begin with 1 and add to 64 by 9's.

17. Begin with 2 and add to 56 by 9's.

18. Begin with 3 and add to 66 by 9's.

WRITTEN EXERCISES.

Copy and add:—

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
9	9	9	9	9	2	9	5	6	7
9	9	9	9	9	9	4	9	6	6
9	9	9	9	9	3	9	9	9	5
9	9	9	9	9	9	4	5	9	4
9	9	9	9	9	5	9	5	6	9
9	9	9	9	9	9	4	9	6	4
9	9	9	9	9	4	9	9	9	5
9	9	9	9	9	9	4	5	9	6
9	9	9	9	9	9	9	5	4	7
9	3	5	7	6	8	4	9	6	8

	11.	**12.**	**13.**	**14.**	**15.**
	$6.54	$24.16	$105.50	$225.00	$400.00
	$5.09	$46.64	$649.85	$108.85	$40.50
Add	$0.76	$8.25	$67.75	$9.88	$4.05

NEW ADDITIVE NUMBER, TEN.

ORAL EXERCISES.

How many are:—

1 and 10?	11 less 10?
2 and 10?	12 less 10?
3 and 10?	13 less 10?
4 and 10?	14 less 10?
5 and 10?	15 less 10?
6 and 10?	16 less 10?
7 and 10?	17 less 10?
8 and 10?	18 less 10?
9 and 10?	19 less 10?
10 and 10?	20 less 10?

Read and complete:—

1 + 10 =	11 — 10 =
2 + 10 =	12 — 10 =
3 + 10 =	13 — 10 =
4 + 10 =	14 — 10 =
5 + 10 =	15 — 10 =
6 + 10 =	16 — 10 =
7 + 10 =	17 — 10 =
8 + 10 =	18 — 10 =
9 + 10 =	19 — 10 =
10 + 10 =	20 — 10 =

1. How many are 7 and 6, less 8? 7 and 8, less 6?
2. How many are 5 and 9, less 7? 6 and 10, less 9?
3. How many are 7 and 8, less 9? 6 and 8, less 7?
4. How many are 7 and 7, less 8? 7 and 6, less 9?
5. How many are 8 and 6, less 7? 8 and 5, less 8?
6. How many are 9 and 6, less 8? 7 and 6, less 10?
7. How many are 5 and 9, less 8? 7 and 8, less 6?
8. How many are 7 and 5, less 8? 9 and 5, less 8?
9. How many are 7+5+6+6—8? 8+4+6+7—6?
10. How many are 6+9+8+7—9? 5+7+8+9—10?
11. Begin with 3 and add to 63 by 6's.
12. Begin with 7 and add to 70 by 7's.
13. Begin with 3 and add to 75 by 8's.
14. Begin with 4 and add to 64 by 6's.
15. Begin with 40 and subtract to 20 by 2's.
16. Begin with 50 and subtract to 0 by 5's.

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	146	239	129	269	70	118	$13.95
	264	93	92	196	89	129	$18.40
	173	149	189	379	60	207	$10.95
	208	194	198	97	299	209	$20.84
	127	59	79	86	140	127	$16.92
Add	109	295	297	68	259	186	$15.50

	8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
From	446	247	358	346	574	666	$5.55
Take	229	119	239	129	249	349	$2.38

	15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
From	250	370	190	293	438	$5.29	$6.07
Take	219	249	169	159	293	$3.95	$3.94

To TEACHERS.—For supplemental exercises in addition and subtraction, see *Oral Lessons in Number*, pp. 111–114 and pp. 155–159.

MULTIPLICATION AND DIVISION.

ORAL EXERCISES.

1. How many times do I say *la*, children: *la*, *la?* How many times now: *la*, *la*, *la?*

2. Speak *la* twice. Speak *la* 3 times; 4 times.

3. Clap your hands once. Clap them twice. Clap them 4 times. Clap them 5 times.

4. How many are 3 times 1 pencil? 5 times 1 pencil? 4 times 1 pencil? 8 times 1 pencil? 6 times 1 pencil?

5. How many are twice 1? 3 times 1? 4 times 1? 5 times 1? 6 times 1? 9 times 1? 10 times 1?

One 1 ?	1 time 1?	1 in 1?
Two 1's?	2 times 1?	1 in 2?
Three 1's?	3 times 1?	1 in 3?
Four 1's?	4 times 1?	1 in 4?
Five 1's?	5 times 1?	1 in 5?
Six 1's?	6 times 1?	1 in 6?
Seven 1's?	7 times 1?	1 in 7?
Eight 1's?	8 times 1?	1 in 8?
Nine 1's?	9 times 1?	1 in 9?
Ten 1's?	10 times 1?	1 in 10?

To Teachers.—These tables may be recited from left to right; thus, three 1's are 3; 3 times 1 is 3; 1 in 3, 3 times. When pupils are familiar with the first table, the second and third tables only should be recited; thus, 3 times 1 is 3; 1 in 3, 3 times.

When pupils can not give the product, let them add the multiplicand number to the next lower product. Thus, the product of 7 times 6 is to be found by adding 6 to the product of 6 times 6.

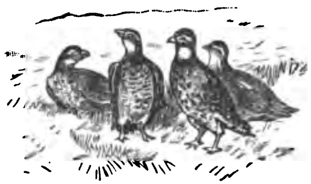
6. How many are 3 times 1? 5 times 1? 7 times 1? 9 times 1? 8 times 0? 6 times 0? 10 times 1?

7. How many times 1 in 3? 1 in 5? 1 in 7? 1 in 9? 1 in 4? 1 in 6? 1 in 8? 1 in 10?

MULTIPLICAND AND DIVISOR, TWO.

ORAL EXERCISES.

1. How many quails are 2 times 2 quails? 3 times 2 quails? 4 times 2 quails?



2. How many quails are 5 times 2 quails? 6 times 2 quails? 7 times 2 quails?

3. How many keys are 3 times 2 keys? 5 times 2 keys? 7 times 2 keys?

4. How many times 2 keys in 4 keys? 2 keys in 6 keys? 2 keys in 8 keys? 2 keys in 10 keys?

5. How many are two 2's? Three 2's? Four 2's? Five 2's? Six 2's? Seven 2's? Eight 2's? Nine 2's? Ten 2's?

One	2 ?	1 time	2?	2 in	2?
Two	2's?	2 times	2?	2 in	4?
Three	2's?	3 times	2?	2 in	6?
Four	2's?	4 times	2?	2 in	8?
Five	2's?	5 times	2?	2 in	10?
Six	2's?	6 times	2?	2 in	12?
Seven	2's?	7 times	2?	2 in	14?
Eight	2's?	8 times	2?	2 in	16?
Nine	2's?	9 times	2?	2 in	18?
Ten	2's?	10 times	2?	2 in	20?

6. How many are 2 times 2? 4 times 2? 6 times 2? 8 times 2? 10 times 2?

7. How many are 3 times 2? 5 times 2? 7 times 2? 9 times 2? Once 2? 6 times 2? 8 times 2?

8. How many 2's in 4? 2's in 8? 2's in 10? 2's in 14? 2's in 12? 2's in 16? 2's in 18? 2's in 20?

9. How many times 2 in 2? 2 in 4? 2 in 6? 2 in 8? 2 in 12? 2 in 6? 2 in 10? 2 in 14? 2 in 18?

10. What is the cost of 6 two-cent postage stamps? 10 two-cent stamps? 8 two-cent stamps?

11. What will 5 oranges cost at 2 cents apiece? 8 oranges? 6 oranges? 9 oranges? 10 oranges?

12. At 2 cents each what will 4 pencils cost? 7 pencils? 10 pencils?

WRITTEN EXERCISES.

Copy and fill:—

$1 \times 2 = 2$
$2 \times 2 =$
$3 \times 2 =$
$4 \times 2 =$
$5 \times 2 =$
$6 \times 2 =$
$7 \times 2 =$
$8 \times 2 =$
$9 \times 2 =$
$10 \times 2 =$

$2 \div 2 = 1$
$4 \div 2 =$
$6 \div 2 =$
$8 \div 2 =$
$10 \div 2 =$
$12 \div 2 =$
$14 \div 2 =$
$16 \div 2 =$
$18 \div 2 =$
$20 \div 2 =$

$3 \times 2 =$
$6 \div 2 =$
$5 \times 2 =$
$10 \div 2 =$
$7 \times 2 =$
$14 \div 2 =$
$8 \times 2 =$
$16 \div 2 =$
$9 \times 2 =$
$18 \div 2 =$

NOTE.—The sign × may be read *time* after 1 and *times* after other integers; thus, 3×2 may be read 3 times 2. The sign ÷ is read *divided by*. $6 \div 2 =$ may be recited 2 *in* 6, 3 *times*.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	3	5	7	8	4	9	10
Multiply	2	2	2	2	2	2	2
	6						

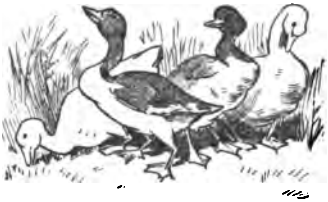
	8.	**9.**	**10.**	**11.**	**12.**	**13.**
Divide	$2\overline{)10}$	$2\overline{)14}$	$2\overline{)16}$	$2\overline{)8}$	$2\overline{)18}$	$2\overline{)20}$

	14.	**15.**	**16.**	**17.**	**18.**	**19.**
	12	22	22	22	222	222
Multiply	2	2	3	4	3	4
	24	44			666	

	20.	**21.**	**22.**	**23.**	**24.**
Divide	$2\overline{)44}$	$2\overline{)66}$	$2\overline{)88}$	$2\overline{)666}$	$2\overline{)888}$

MULTIPLICAND AND DIVISOR, THREE.

ORAL EXERCISES.



1. How many geese are twice 2 geese? 3 times 2 geese? 8 times 2 geese?

2. How many are 4 times 2 geese? 5 times 2 geese? 6 times 2 geese? 7 times 2 geese? 8 times 2 geese?

3. How many books are 3 times 3 books? 4 times 3 books? 6 times 3 books? 8 times 3 books? 9 times 3 books? 10 times 3 books?

4. How many times 3 books in 6 books? 3 books in 9 books? 3 books in 12 books? 3 books in 15 books?

5. How many are two 3's? Three 3's? Four 3's? Five 3's? Six 3's? Seven 3's? Eight 3's? Nine 3's? Ten 3's?

One 3?	1 time 3?	3 in 3?
Two 3's?	2 times 3?	3 in 6?
Three 3's?	3 times 3?	3 in 9?
Four 3's?	4 times 3?	3 in 12?
Five 3's?	5 times 3?	3 in 15?
Six 3's?	6 times 3?	3 in 18?
Seven 3's?	7 times 3?	3 in 21?
Eight 3's?	8 times 3?	3 in 24?
Nine 3's?	9 times 3?	3 in 27?
Ten 3's?	10 times 3?	3 in 30?

6. How many are 3 times 3? 5 times 3? 6 times 3? 7 times 3? 9 times 3? 10 times 3?

7. How many are twice 3? 4 times 3? 6 times 3? 8 times 3? 10 times 3? Once 3?

8. How many 3's in 6? 3's in 9? 3's in 15? 3's in 21? 3's in 24? 3's in 18? 3's in 27?

9. If a barrel will hold 3 bushels of apples, how many bushels will 7 barrels hold?

10. A drover bought 9 sheep at $3 a head: how many dollars did they cost?

11. What will 6 pencils cost at 3 cents each? 5 pencils? 7 pencils? 10 pencils?

12. There are 3 feet in a yard: how many feet in 6 yards? In 8 yards? 10 yards? 5 yards?

WRITTEN EXERCISES.

Copy and fill:—

$1 \times 3 = 3$
$2 \times 3 =$
$3 \times 3 =$
$4 \times 3 =$
$5 \times 3 =$
$6 \times 3 =$
$7 \times 3 =$
$8 \times 3 =$
$9 \times 3 =$
$10 \times 3 =$

$3 \div 3 = 1$
$6 \div 3 =$
$9 \div 3 =$
$12 \div 3 =$
$15 \div 3 =$
$18 \div 3 =$
$21 \div 3 =$
$24 \div 3 =$
$27 \div 3 =$
$30 \div 3 =$

$2 \times 3 =$
$6 \div 3 =$
$4 \times 3 =$
$12 \div 3 =$
$6 \times 3 =$
$18 \div 3 =$
$8 \times 3 =$
$24 \div 3 =$
$10 \times 3 =$
$30 \div 3 =$

$9 \div 3 =$
$12 \div 3 =$
$18 \div 3 =$
$24 \div 3 =$
$30 \div 3 =$
$21 \div 3 =$
$27 \div 3 =$
$15 \div 3 =$
$9 \times 3 =$
$7 \times 3 =$

	1.	**2.**	**3.**	**4.**	**5.**
	30	30	30	30	30
Multiply	3	4	6	5	7
	90, *Ans.*	120			

	6.	**7.**	**8.**	**9.**	**10.**
	31	31	31	31	31
Multiply	4	5	8	9	6

	11.	**12.**	**13.**	**14.**	**15.**	**16.**
	32	32	32	32	32	32
Multiply	4	3	6	7	8	9
			192			

MULTIPLICAND AND DIVISOR, FOUR.

ORAL EXERCISES.

1. How many arrows are twice 4 arrows? 3 times 4 arrows? 4 times 4 arrows?

2. How many arrows are 5 times 4 arrows? 6 times 4 arrows? 7 times 4 arrows?

3. How many wheels are 4 times 4 wheels? 8 times 4 wheels? 9 times 4 wheels? 10 times 4 wheels?

4. How many times 4 eggs in 16 eggs? 4 eggs in 12 eggs? 4 eggs in 20 eggs? 4 eggs in 24 eggs?

5. How many are 5 times 4 dimes? 6 times 4 dimes? 7 times 4 dimes? 10 times 4 dimes?

6. How many are twice 4 cents? 3 times 4 cents? 4 times 4 cents? 5 times 4 cents? 6 times 4 cents?

7. How many are three 4's? Four 4's? Five 4's? Six 4's? Seven 4's? Eight 4's? Nine 4's? Ten 4's?

One 4?	1 time 4?	4 in 4?
Two 4's?	2 times 4?	4 in 8?
Three 4's?	3 times 4?	4 in 12?
Four 4's?	4 times 4?	4 in 16?
Five 4's?	5 times 4?	4 in 20?
Six 4's?	6 times 4?	4 in 24?
Seven 4's?	7 times 4?	4 in 28?
Eight 4's?	8 times 4?	4 in 32?
Nine 4's?	9 times 4?	4 in 36?
Ten 4's?	10 times 4?	4 in 40?

8. How many are 3 times 4? 5 times 4? 6 times 4? 8 times 4? 9 times 4? 10 times 4?

9. How many are twice 4? 4 times 4? 6 times 4? 9 times 4? 5 times 4? 7 times 4?

10. How many times 4 in 8? 4 in 16? 4 in 24? 4 in 32? 4 in 40? 4 in 20? 4 in 36?

11. How many 4's in 12? 4's in 20? 4's in 28? 4's in 32? 4's in 40? 4's in 36?

12. There are 4 pecks in a bushel: how many pecks in 5 bushels? In 8 bushels? In 10 bushels?

13. There are 4 quarts in a gallon: how many gallons in 16 quarts? In 24 quarts? In 36 quarts?

14. What will 8 loads of wood cost at $4 a load?

WRITTEN EXERCISES.

Copy and fill:—

$1 \times 4 = 4$
$2 \times 4 =$
$3 \times 4 =$
$4 \times 4 =$
$5 \times 4 =$
$6 \times 4 =$
$7 \times 4 =$
$8 \times 4 =$
$9 \times 4 =$
$10 \times 4 =$

$4 \div 4 = 1$
$8 \div 4 =$
$12 \div 4 =$
$16 \div 4 =$
$20 \div 4 =$
$24 \div 4 =$
$28 \div 4 =$
$32 \div 4 =$
$36 \div 4 =$
$40 \div 4 =$

$2 \times 4 =$
$8 \div 4 =$
$4 \times 4 =$
$16 \div 4 =$
$6 \times 4 =$
$24 \div 4 =$
$8 \times 4 =$
$32 \div 4 =$
$10 \times 4 =$
$40 \div 4 =$

$12 \div 4 =$
$20 \div 4 =$
$28 \div 4 =$
$7 \times 4 =$
$9 \times 4 =$
$10 \times 4 =$
$40 \div 4 =$
$36 \div 4 =$
$24 \div 4 =$
$32 \div 4 =$

	1.	**2.**	**3.**	**4.**	**5.**
	24	34	44	43	42
Multiply	5	6	4	8	9

	6.	**7.**	**8.**	**9.**
Divide	$4\overline{)88}$	$4\overline{)48}$	$4\overline{)80}$	$4\overline{)84}$
	22			

	10.	**11.**	**12.**	**13.**	**14.**
	203	104	114	244	233
Multiply	4	8	7	3	4

	15.	**16.**	**17.**	**18.**	**19.**
Divide	$4\overline{)480}$	$4\overline{)840}$	$4\overline{)808}$	$4\overline{)880}$	$4\overline{)408}$

MULTIPLICAND AND DIVISOR, FIVE.

ORAL EXERCISES.

1. The leaves of a flower are called *petals*. A violet has 5 petals: how many petals have 2 violets? 3 violets? 4 violets? 5 violets? 6 violets? 10 violets?

2. How many are twice 5? 3 times 5? 4 times 5? 5 times 5? 6 times 5? 7 times 5? 8 times 5?

3. There are 5 cents in a nickel: how many cents in 5 nickels? In 6 nickels? In 7 nickels?

4. How many cents are 5 times 5 cents? 7 times 5 cents? 9 times 5 cents? 8 times 5 cents?

One 5 ?	1 time 5?	5 in 5?
Two 5's?	2 times 5?	5 in 10?
Three 5's?	3 times 5?	5 in 15?
Four 5's?	4 times 5?	5 in 20?
Five 5's?	5 times 5?	5 in 25?
Six 5's?	6 times 5?	5 in 30?
Seven 5's?	7 times 5?	5 in 35?
Eight 5's?	8 times 5?	5 in 40?
Nine 5's?	9 times 5?	5 in 45?
Ten 5's?	10 times 5?	5 in 50?

5. How many are 3 times 5? 5 times 5? 9 times 5? 7 times 5? 6 times 5? 8 times 5? 10 times 5?

6. How many times 5 in 15? 5 in 25? 5 in 35? 5 in 45? 5 in 50? 5 in 40?

7. How many are two 5's? Four 5's? Three 5's? Five 5's? Seven 5's? Eight 5's? Six 5's? Nine 5's? Ten 5's?

8. How many 5's in 10? 5's in 20? 5's in 30? 5's in 50? 5's in 40? 5's in 45? 5's in 35?

9. An orchard has 8 rows of trees, and each row has 5 trees: how many trees in the orchard?

10. A school-room has 7 rows of desks, and there are 5 desks in each row: how many desks in the room?

11. What will 6 pencils cost at 5 cents apiece?

12. What will 10 oranges cost at 5 cents apiece?

13. What will 9 hats cost at $5 apiece?

WRITTEN EXERCISES.

Copy and fill:—

1 × 5 = 5
2 × 5 =
3 × 5 =
4 × 5 =
5 × 5 =
6 × 5 =
7 × 5 =
8 × 5 =
9 × 5 =
10 × 5 =

5 ÷ 5 = 1
10 ÷ 5 =
15 ÷ 5 =
20 ÷ 5 =
25 ÷ 5 =
30 ÷ 5 =
35 ÷ 5 =
40 ÷ 5 =
45 ÷ 5 =
50 ÷ 5 =

6 × 5 =
30 ÷ 5 =
7 × 5 =
35 ÷ 5 =
10 × 5 =
50 ÷ 5 =
5 × 5 =
25 ÷ 5 =
9 × 5 =
45 ÷ 5 =

5 × 9 =
40 ÷ 5 =
36 ÷ 4 =
4 × 8 =
30 ÷ 5 =
27 ÷ 3 =
3 × 8 =
50 ÷ 5 =
45 ÷ 5 =
28 ÷ 4 =

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	25	35	45	54	53	52
Multiply	4	7	8	6	7	9

	7.	**8.**	**9.**	**10.**	**11.**
	125	145	254	205	115
Multiply	6	5	3	4	8

	12.	**13.**	**14.**	**15.**	**16.**
Divide	5)255	5)305	5)405	5)455	5)525

	17.	**18.**	**19.**	**20.**	**21.**
Divide	5)125	5)265	5)605	5)705	5)315

MULTIPLICAND AND DIVISOR, SIX.

ORAL EXERCISES.

1. A fly has 6 legs: how many legs have 2 flies? 3 flies? 4 flies?

2. How many legs have 5 flies? 6 flies? 7 flies? 8 flies? 9 flies? 10 flies?

3. How many are twice 6? 3 times 6? 4 times 6? 5 times 6? 6 times 6? 8 times 6? 10 times 6?

4. How many spools are 7 times 6 spools? 8 times 6 spools? 9 times 6 spools? 10 times 6 spools?

5. How many are two 6's? Three 6's? Four 6's? Five 6's? Six 6's? Seven 6's? Eight 6's? Nine 6's? Ten 6's?

One 6 ?	1 time 6?	6 in 6?
Two 6's?	2 times 6?	6 in 12?
Three 6's?	3 times 6?	6 in 18?
Four 6's?	4 times 6?	6 in 24?
Five 6's?	5 times 6?	6 in 30?
Six 6's?	6 times 6?	6 in 36?
Seven 6's?	7 times 6?	6 in 42?
Eight 6's?	8 times 6?	6 in 48?
Nine 6's?	9 times 6?	6 in 54?
Ten 6's?	10 times 6?	6 in 60?

6. How many are twice 6? 4 times 6? 6 times 6? 8 times 6? 10 times 6? 5 times 6? 7 times 6?

7. How many are 3 times 6? 5 times 6? 7 times 6? 9 times 6? 8 times 6? 10 times 6? 6 times 6?

8. How many 6's in 18? 6's in 30? 6's in 42? 6's in 60? 6's in 54? 6's in 36? 6's in 48?

9. How many times 6 in 24? 6 in 36? 6 in 18? 6 in 60? 6 in 30? 6 in 42? 6 in 48? 6 in 54?

10. Mary wrote 5 columns of figures, and each column had 6 figures: how many figures did she write?

11. What will 6 oranges cost, at 6 cents apiece?

12. A school-house has 9 windows, and each window has 6 panes of glass: how many panes of glass in the house?

WRITTEN EXERCISES.

Copy and fill:—

$1 \times 6 = 6$
$2 \times 6 =$
$3 \times 6 =$
$4 \times 6 =$
$5 \times 6 =$
$6 \times 6 =$
$7 \times 6 =$
$8 \times 6 =$
$9 \times 6 =$
$10 \times 6 =$

$6 \div 6 = 1$
$12 \div 6 =$
$18 \div 6 =$
$24 \div 6 =$
$30 \div 6 =$
$36 \div 6 =$
$42 \div 6 =$
$48 \div 6 =$
$54 \div 6 =$
$60 \div 6 =$

$6 \times 7 =$
$42 \div 6 =$
$6 \times 9 =$
$54 \div 6 =$
$6 \times 8 =$
$48 \div 6 =$
$10 \times 6 =$
$60 \div 6 =$
$5 \times 6 =$
$30 \div 6 =$

$6 \times 7 = 42$
$5 \times \quad = 40$
$6 \times \quad = 54$
$6 \times \quad = 36$
$5 \times \quad = 50$
$6 \times \quad = 48$
$4 \times \quad = 36$
$5 \times \quad = 35$
$5 \times \quad = 45$
$6 \times \quad = 60$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	26	36	46	56	66	63
Multiply	7	8	9	8	6	7

	7.	**8.**	**9.**	**10.**	**11.**
	136	246	106	226	135
Multiply	5	4	9	3	6

	12.	**13.**	**14.**	**15.**	**16.**
Divide	6)426	6)546	6)660	6)306	6)480

	17.	**18.**	**19.**	**20.**	**21.**
Divide	6)372	6)264	6)330	6)444	6)522

	22.	**23.**	**24.**	**25.**	**26.**
Divide	6)498	6)384	6)564	6)732	6)852

MULTIPLICAND AND DIVISOR, SEVEN.

ORAL EXERCISES.

1. This rose leaf has 7 blades: how many blades have 2 leaves?

2. How many blades are 3 times 7 blades? 4 times 7 blades? 5 times 7 blades? 6 times 7 blades?

3. How many are twice 7 leaves? 3 times 7 leaves? 4 times 7 leaves? 5 times 7 leaves? 6 times 7 leaves?

4. How many grapes are 7 times 7 grapes? 8 times 7 grapes? 9 times 7 grapes? 10 times 7 grapes? 4 times 7 grapes? 5 times 7 grapes?

5. How many are three 7's? Four 7's? Five 7's? Six 7's? Seven 7's? Eight 7's? Nine 7's? Ten 7's?

One 7 ?	1 time 7?	7 in 7?
Two 7's?	2 times 7?	7 in 14?
Three 7's?	3 times 7?	7 in 21?
Four 7's?	4 times 7?	7 in 28?
Five 7's?	5 times 7?	7 in 35?
Six 7's?	6 times 7?	7 in 42?
Seven 7's?	7 times 7?	7 in 49?
Eight 7's?	8 times 7?	7 in 56?
Nine 7's?	9 times 7?	7 in 63?
Ten 7's?	10 times 7?	7 in 70?

6. How many are 6 times 7? 7 times 7? 8 times 7? 9 times 7? 10 times 7? 4 times 7? 3 times 7?

7. How many are twice 7? 4 times 7? 6 times 7? 8 times 7? 10 times 7? 7 times 7? 5 times 7?

8. How many times 7 in 14? 7 in 21? 7 in 35? 7 in 42? 7 in 49? 7 in 56? 7 in 63?

9. How many 7's in 14? 7's in 21? 7's in 35? 7's in 28? 7's in 56? 7's in 63? 7's in 49? 7's in 70?

10. There are 7 days in a week: how many days in 7 weeks? How many days in 9 weeks?

11. There are 7 days in a week: how many weeks in 28 days? In 56 days? In 49 days?

12. What will 8 quarts of milk cost at 7 cents a quart? 10 quarts? 6 quarts?

13. At 7 cents a quart, how many quarts of milk can be bought for 63 cents? For 56 cents?

WRITTEN EXERCISES.

Copy and fill:—

$1 \times 7 =$
$2 \times 7 =$
$3 \times 7 =$
$4 \times 7 =$
$5 \times 7 =$
$6 \times 7 =$
$7 \times 7 =$
$8 \times 7 =$
$9 \times 7 =$
$10 \times 7 =$

$7 \div 7 =$
$14 \div 7 =$
$21 \div 7 =$
$28 \div 7 =$
$35 \div 7 =$
$42 \div 7 =$
$49 \div 7 =$
$56 \div 7 =$
$63 \div 7 =$
$70 \div 7 =$

$4 \times 7 =$
$28 \div 7 =$
$6 \times 7 =$
$42 \div 7 =$
$8 \times 7 =$
$56 \div 7 =$
$9 \times 7 =$
$63 \div 7 =$
$7 \times 7 =$
$49 \div 7 =$

$35 \div \quad = 7$
$42 \div \quad = 7$
$28 \div \quad = 7$
$49 \div \quad = 7$
$56 \div \quad = 8$
$63 \div \quad = 9$
$28 \div \quad = 4$
$42 \div \quad = 6$
$56 \div \quad = 7$
$70 \div \quad = 7$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	37	47	73	74	75	76	77
Multiply	8	9	6	7	8	7	9

	8.	**9.**	**10.**	**11.**	**12.**	**13.**
	137	176	207	127	107	77
Multiply	6	5	4	7	8	9

	14.	**15.**	**16.**	**17.**	**18.**
Divide	8)120	8)168	8)208	8)272	8)264

	19.	**20.**	**21.**	**22.**	**23.**
Divide	6)252	7)364	8)280	5)375	4)380

MULTIPLICAND AND DIVISOR, EIGHT.

ORAL EXERCISES.

1. There are 8 panes of glass in a window: how many panes in 2 windows? In 3 windows?

2. How many are 3 times 8 panes? 4 times 8 panes? 5 times 8 panes? 6 times 8 panes?

3. How many are 2 times 8 pins? 3 times 8 pins? 4 times 8 pins? 5 times 8 pins?

4. How many pins are 6 times 8 pins? 7 times 8 pins? 9 times 8 pins? 10 times 8 pins?

5. How many are two 8's? Four 8's? Five 8's? Six 8's? Seven 8's? Eight 8's? Nine 8's? Ten 8's?

One 8 ?	1 time 8?	8 in 8?
Two 8's?	2 times 8?	8 in 16?
Three 8's?	3 times 8?	8 in 24?
Four 8's?	4 times 8?	8 in 32?
Five 8's?	5 times 8?	8 in 40?
Six 8's?	6 times 8?	8 in 48?
Seven 8's?	7 times 8?	8 in 56?
Eight 8's?	8 times 8?	8 in 64?
Nine 8's?	9 times 8?	8 in 72?
Ten 8's?	10 times 8?	8 in 80?

6. How many are 3 times 8? 5 times 8? 7 times 8? 9 times 8? 6 times 8? 4 times 8?

7. How many are 4 times 8? 6 times 8? 8 times 8? 10 times 8? 7 times 8? 9 times 8?

8. How many times 8 in 24? 8 in 40? 8 in 56? 8 in 72? 8 in 80? 8 in 0? 8 in 16?

9. How many times 8 in 32? 8 in 48? 8 in 64? 8 in 80? 8 in 56? 8 in 40?

10. How many 8's in 16? 8's in 40? 8's in 32? 8's in 56? 8's in 24? 8's in 64? 8's in 80? 8's in 72?

11. There are 8 quarts in a peck: how many quarts in 3 pecks? 5 pecks? 8 pecks? 6 pecks? 9 pecks?

12. If 8 quarts are a peck, how many pecks in 16 quarts? In 24 quarts? In 40 quarts?

13. At 8 cents apiece, how much will 4 copy-books cost? 7 copy-books? 10 copy-books?

14. At 8 cents a yard, how many yards of muslin can be bought for 40 cents? For 80 cents?

WRITTEN EXERCISES.

Copy and fill:—

1 × 8 =
2 × 8 =
3 × 8 =
4 × 8 =
5 × 8 =
6 × 8 =
7 × 8 =
8 × 8 =
9 × 8 =
10 × 8 =

8 ÷ 8 =
16 ÷ 8 =
24 ÷ 8 =
32 ÷ 8 =
40 ÷ 8 =
48 ÷ 8 =
56 ÷ 8 =
64 ÷ 8 =
72 ÷ 8 =
80 ÷ 8 =

6 × 8 =
48 ÷ 8 =
8 × 8 =
64 ÷ 8 =
5 × 8 =
40 ÷ 8 =
9 × 8 =
72 ÷ 8 =
7 × 8 =
56 ÷ 8 =

7 × = 56
8 × = 72
7 × = 42
6 × = 54
7 × = 63
8 × = 56
8 × = 80
5 × = 45
6 × = 48
10 × = 70

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	28	48	88	68	78	58
Multiply	8	7	6	9	7	8

	7.	**8.**	**9.**	**10.**	**11.**
	$138	$217	$178	$108	$117
Multiply	6	4	5	9	8

	12.	**13.**	**14.**	**15.**	**16.**
Divide	8)168	8)288	8)208	8)272	8)264

	17.	**18.**	**19.**	**20.**	**21.**
Divide	8)$456	8)$664	8)$768	8)$448	8)$504

MULTIPLICAND AND DIVISOR, NINE.

ORAL EXERCISES.

1. There are 9 ears of corn in a bunch: how many ears in 2 bunches? In 3 bunches?

2. How many ears are twice 9 ears? 3 times 9 ears? 4 times 9 ears? 5 times 9 ears?

3. How many are 2 times 9? 3 times 9? 4 times 9? 5 times 9?

4. How many are 6 times 9 trees? 7 times 9 trees? 8 times 9 trees? 9 times 9 trees? 10 times 9 trees? 5 times 9 trees?

5. How many are two 9's? Three 9's? Four 9's? Five 9's? Six 9's? Seven 9's? Eight 9's? Nine 9's?

One 9?	1 time 9?	9 in 9?
Two 9's?	2 times 9?	9 in 18?
Three 9's?	3 times 9?	9 in 27?
Four 9's?	4 times 9?	9 in 36?
Five 9's?	5 times 9?	9 in 45?
Six 9's?	6 times 9?	9 in 54?
Seven 9's?	7 times 9?	9 in 63?
Eight 9's?	8 times 9?	9 in 72?
Nine 9's?	9 times 9?	9 in 81?
Ten 9's?	10 times 9?	9 in 90?

6. How many are 3 times 9? 5 times 9? 7 times 9? 4 times 9? 6 times 9? 8 times 9? 10 times 9?

7. How many are 4 times 9? 6 times 9? 8 times 9? 10 times 9? 9 times 7? 9 times 6? 9 times 8?

8. How many times 9 in 36? 9 in 45? 9 in 27? 9 in 54? 9 in 72? 9 in 18? 9 in 9?

9. How many times 9 in 63? 9 in 72? 9 in 90? 9 in 36? 9 in 54? 9 in 81? 9 in 45?

10. How many 9's in 18? 9's in 36? 9's in 54? 9's in 72? 9's in 90? 9's in 27? 9's in 45? 9's in 81?

11. There are 7 rows of trees in a peach orchard, and 9 trees in each row: how many trees in the orchard?

12. A farmer filled 8 bags with wheat, putting 9 pecks in each bag: how many pecks of wheat in the bags?

13. A drover bought 10 hogs at $9 apiece: how much did he pay for them?

14. What will 6 heads of cabbage cost at 9 cents a head? What will 10 heads cost?

WRITTEN EXERCISES.

Copy and fill:—

1 × 9 =
2 × 9 =
3 × 9 =
4 × 9 =
5 × 9 =
6 × 9 =
7 × 9 =
8 × 9 =
9 × 9 =
10 × 9 =

9 ÷ 9 =
18 ÷ 9 =
27 ÷ 9 =
36 ÷ 9 =
45 ÷ 9 =
54 ÷ 9 =
63 ÷ 9 =
72 ÷ 9 =
81 ÷ 9 =
90 ÷ 9 =

7 × 9 =
63 ÷ 9 =
9 × 9 =
81 ÷ 9 =
5 × 9 =
45 ÷ 9 =
8 × 9 =
72 ÷ 9 =
10 × 9 =
90 ÷ 9 =

6 × = 42
6 × = 54
7 × = 56
7 × = 63
8 × = 72
8 × = 80
9 × = 54
9 × = 72
9 × = 63
9 × = 81

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	$49	$69	$89	$39	$59	$79
Multiply	5	4	3	7	9	6

	7.	**8.**	**9.**	**10.**	**11.**	**12.**
	149	169	189	139	109	99
Multiply	5	4	3	7	9	8

	13.	**14.**	**15.**	**16.**	**17.**
Divide	9)$189	9)$288	9)$396	9)$567	9)$585

	18.	**19.**	**20.**	**21.**	**22.**
Divide	9)747	9)846	9)774	9)693	9)684

MULTIPLICAND AND DIVISOR, TEN.

ORAL EXERCISES.

1. There are 10 cents in one dime: how many cents in 2 dimes? In 3 dimes? 4 dimes? 5 dimes? 6 dimes?

2. How many are 6 times 10 dimes? 7 times 10 dimes? 8 times 10 dimes? 9 times 10 dimes? 10 times 10 dimes?

3. How many are two 10's? Three 10's? Four 10's? Five 10's? Six 10's? Seven 10's? Eight 10's? Nine 10's? Ten 10's?

One 10 ?	1 time 10?	10 in 10?
Two 10's?	2 times 10?	10 in 20?
Three 10's?	3 times 10?	10 in 30?
Four 10's?	4 times 10?	10 in 40?
Five 10's?	5 times 10?	10 in 50?
Six 10's?	6 times 10?	10 in 60?
Seven 10's?	7 times 10?	10 in 70?
Eight 10's?	8 times 10?	10 in 80?
Nine 10's?	9 times 10?	10 in 90?
Ten 10's?	10 times 10?	10 in 100?

4. How many are 5 times 10? 7 times 10? 9 times 10? 8 times 10? 6 times 10? 4 times 10?

5. How many times 10 in 20? 10 in 30? 10 in 50? 10 in 40? 10 in 60? 10 in 70? 10 in 90? 10 in 80?

6. How many 10's in 70? 10's in 80? 10's in 100? 10's in 60? 10's in 50? 10's in 40? 10's in 90?

7. There are 10 cents in 1 dime: how many cents in 6 dimes? In 8 dimes? 7 dimes? 10 dimes?

8. There are 10 dimes in 1 dollar: how many dimes in 4 dollars? In 7 dollars? 5 dollars? 8 dollars?

9. How many dimes in 40 cents? In 60 cents? 80 cents? 100 cents? 50 cents? 70 cents?

10. Here is a woman selling postage-stamps. The sheet in her hand has 10 rows of two-cent stamps and 10 stamps in each row: how many stamps in the sheet?

11. What will be the cost of one row of the stamps? Of 2 rows? Of 5 rows? Of the entire sheet?

WRITTEN EXERCISES.

Copy and fill:—

1 × 10 =
2 × 10 =
3 × 10 =
4 × 10 =
5 × 10 =
6 × 10 =
7 × 10 =
8 × 10 =
9 × 10 =
10 × 10 =

10 ÷ 10 =
20 ÷ 10 =
30 ÷ 10 =
40 ÷ 10 =
50 ÷ 10 =
60 ÷ 10 =
70 ÷ 10 =
80 ÷ 10 =
90 ÷ 10 =
100 ÷ 10 =

7 × 9 =
9 × 8 =
7 × 8 =
8 × 6 =
10 × 7 =
10 × 9 =
6 × 9 =
9 × 7 =
6 × 8 =
8 × 9 =

63 ÷ = 9
72 ÷ = 9
81 ÷ = 9
90 ÷ = 9
64 ÷ = 8
80 ÷ = 8
63 ÷ = 7
56 ÷ = 7
70 ÷ = 7
49 ÷ = 7

	1.	**2.**	**3.**	**4.**	**5.**
Divide	5)450	5)665	5)725	6)174	6)906
	6.	**7.**	**8.**	**9.**	**10.**
Divide	7)231	7)266	7)476	7)189	7)329
	11.	**12.**	**13.**	**14.**	**15.**
Divide	8)272	8)408	8)512	8)328	8)504
	16.	**17.**	**18.**	**19.**	**20.**
Divide	9)279	9)396	9)477	9)648	9)864

EQUAL PARTS OF NUMBERS.

1. If a melon be cut into two *equal* pieces, what part of the melon will each piece be?

2. If a melon be cut into four equal pieces, what part will each piece be?

3. If an apple be divided into three equal parts, what is each part called?

4. What is one of the *five* equal parts of an object called? One of the *six* equal parts?

5. What is one half of 6 eggs? One half of 16 eggs? One third of 9 eggs? One third of 15 eggs?

6. What is one fourth of 8 cents? One fourth of 16 cents? One fourth of 28 cents? Of 40 cents?

7. What is one fourth of 20? One fourth of 28? One fourth of 36? One fourth of 40?

8. How much is one fifth of 15? One fifth of 25? One fifth of 35? One fifth of 40? One fifth of 50?

9. How much is one sixth of 24? One sixth of 36? One sixth of 48? One sixth of 42? One sixth of 60?

10. How much is one seventh of 21? One seventh of 35? One seventh of 56? One seventh of 42?

11. How much is one eighth of 32? One eighth of 56? One eighth of 72? One eighth of 80?

12. How much is one ninth of 36? One ninth of 63? One ninth of 81? One ninth of 72?

13. How much is one tenth of 30? One tenth of 60?

14. How much is one sixth of $12? One seventh of $42? One eighth of $40? One ninth of $63? One tenth of $100?

15. How much is one fourth of 20 cents? Two fourths of 20 cents? Three fourths of 20 cents?

16. How much is one fifth of 25 cents? Two fifths of 25 cents? Four fifths of 25 cents?

REVIEW EXERCISES.

Read and complete:—

1. $12 = 4 \times$ $12 = 3 \times$ $12 \div 4 =$ $12 \div 3 =$
2. $24 = 6 \times$ $24 = 4 \times$ $24 \div 6 =$ $24 \div 4 =$
3. $21 = 7 \times$ $21 = 3 \times$ $21 \div 7 =$ $21 \div 3 =$
4. $20 = 5 \times$ $20 = 4 \times$ $20 \div 5 =$ $20 \div 4 =$
5. $30 = 6 \times$ $30 = 5 \times$ $30 \div 6 =$ $30 \div 5 =$
6. $54 = 9 \times$ $54 = 6 \times$ $54 \div 9 =$ $54 \div 6 =$
7. $45 = 9 \times$ $45 = 5 \times$ $45 \div 9 =$ $45 \div 5 =$
8. $56 = 8 \times$ $56 = 7 \times$ $56 \div 8 =$ $56 \div 7 =$
9. $42 = 7 \times$ $42 = 6 \times$ $42 \div 7 =$ $42 \div 6 =$
10. $72 = 9 \times$ $72 = 8 \times$ $72 \div 9 =$ $72 \div 8 =$
11. $63 = 9 \times$ $63 = 7 \times$ $63 \div 9 =$ $63 \div 7 =$
12. $70 = 10 \times$ $70 = 7 \times$ $70 \div 10 =$ $70 \div 7 =$
13. $90 = 10 \times$ $90 = 9 \times$ $90 \div 10 =$ $90 \div 9 =$

BLACKBOARD EXERCISES.

1. $5 + 8, - 8, \times 8, \div 8 =$
$5 + 9, - 9, \times 9, \div 9 =$
$7 + 9, - 9, \times 9, \div 9 =$
$7 + 8, - 8, \times 8, \div 8 =$
$8 + 9, - 9, \times 9, \div 9 =$
$9 + 8, - 8, \times 8, \div 8 =$

2. $7 + 7, - 7, \times 7, \div 7 =$
$8 + 8, - 8, \times 8, \div 8 =$
$8 + 6, - 6, \times 6, \div 6 =$
$9 + 7, - 7, \times 7, \div 7 =$
$9 + 9, - 9, \times 9, \div 9 =$
$9 + 6, - 6, \times 6, \div 6 =$

3. $5 + 7, - 4, \times 7, + 4, \div 6, \div 2, \times 9, \div 5 =$ what?
4. $9 \times 6, + 7, - 5, \div 8, + 5, \times 6, + 9, \div 9 =$ what?
5. $62 - 6, \div 8, \times 6, + 9, - 6, \div 9, + 4, \times 8 =$ what?
6. $56 \div 7, \times 5, + 8, - 3, \div 9, \times 4, \times 3 \div 6 =$ what?
7. $75 + 6, \div 9, \times 8, + 8, \div 8, + 3, + 7, \times 6 =$ what?
8. $81 \div 9, \times 7, + 7, \div 10, \times 8, + 4, \div 6, \times 9 =$ what?

NOTE.—The comma (,) is used in the above exercises to indicate that the operations expressed by the signs +, —, ×, and ÷, are to be performed *in the order in which they occur from left to right.* If only + and — were used, the comma would not be needed. See *Oral Lessons in Number,* pages 153 and 160.

Multiplication Table.

1 × 1 = 1	1 × 2 = 2	1 × 3 = 3	1 × 4 = 4
2 × 1 = 2	2 × 2 = 4	2 × 3 = 6	2 × 4 = 8
3 × 1 = 3	3 × 2 = 6	3 × 3 = 9	3 × 4 = 12
4 × 1 = 4	4 × 2 = 8	4 × 3 = 12	4 × 4 = 16
5 × 1 = 5	5 × 2 = 10	5 × 3 = 15	5 × 4 = 20
6 × 1 = 6	6 × 2 = 12	6 × 3 = 18	6 × 4 = 24
7 × 1 = 7	7 × 2 = 14	7 × 3 = 21	7 × 4 = 28
8 × 1 = 8	8 × 2 = 16	8 × 3 = 24	8 × 4 = 32
9 × 1 = 9	9 × 2 = 18	9 × 3 = 27	9 × 4 = 36
10 × 1 = 10	10 × 2 = 20	10 × 3 = 30	10 × 4 = 40
11 × 1 = 11	11 × 2 = 22	11 × 3 = 33	11 × 4 = 44
12 × 1 = 12	12 × 2 = 24	12 × 3 = 36	12 × 4 = 48
1 × 5 = 5	1 × 6 = 6	1 × 7 = 7	1 × 8 = 8
2 × 5 = 10	2 × 6 = 12	2 × 7 = 14	2 × 8 = 16
3 × 5 = 15	3 × 6 = 18	3 × 7 = 21	3 × 8 = 24
4 × 5 = 20	4 × 6 = 24	4 × 7 = 28	4 × 8 = 32
5 × 5 = 25	5 × 6 = 30	5 × 7 = 35	5 × 8 = 40
6 × 5 = 30	6 × 6 = 36	6 × 7 = 42	6 × 8 = 48
7 × 5 = 35	7 × 6 = 42	7 × 7 = 49	7 × 8 = 56
8 × 5 = 40	8 × 6 = 48	8 × 7 = 56	8 × 8 = 64
9 × 5 = 45	9 × 6 = 54	9 × 7 = 63	9 × 8 = 72
10 × 5 = 50	10 × 6 = 60	10 × 7 = 70	10 × 8 = 80
11 × 5 = 55	11 × 6 = 66	11 × 7 = 77	11 × 8 = 88
12 × 5 = 60	12 × 6 = 72	12 × 7 = 84	12 × 8 = 96
1 × 9 = 9	1 × 10 = 10	1 × 11 = 11	1 × 12 = 12
2 × 9 = 18	2 × 10 = 20	2 × 11 = 22	2 × 12 = 24
3 × 9 = 27	3 × 10 = 30	3 × 11 = 33	3 × 12 = 36
4 × 9 = 36	4 × 10 = 40	4 × 11 = 44	4 × 12 = 48
5 × 9 = 45	5 × 10 = 50	5 × 11 = 55	5 × 12 = 60
6 × 9 = 54	6 × 10 = 60	6 × 11 = 66	6 × 12 = 72
7 × 9 = 63	7 × 10 = 70	7 × 11 = 77	7 × 12 = 84
8 × 9 = 72	8 × 10 = 80	8 × 11 = 88	8 × 12 = 96
9 × 9 = 81	9 × 10 = 90	9 × 11 = 99	9 × 12 = 108
10 × 9 = 90	10 × 10 = 100	10 × 11 = 110	10 × 12 = 120
11 × 9 = 99	11 × 10 = 110	11 × 11 = 121	11 × 12 = 132
12 × 9 = 108	12 × 10 = 120	12 × 11 = 132	12 × 12 = 144

NOTE.—The sign ×, in the above table, is to be read *time* when it follows 1, and *times* when it follows other numbers.

PART TWO.

NUMERATION AND NOTATION.

UNITS' PERIOD.

WHEN a number is expressed by two figures, the first or right-hand figure denotes *units*, and the second figure denotes *tens*. Thus, in 25, the 2 denotes tens, and the 5 denotes units.

1. How many tens and how many units in 37? In 57? 64? 88? 94? 78? 80? 90?

In reading a number expressed by two figures, the tens and units are read together as so many units. Thus, 45 is read *forty-five units*, or, more briefly, *forty-five.*

2. Read 29; 48; 76; 60; 84; 90; 67; 58; 75.

When a number is expressed by three figures, the third or left-hand figure denotes *hundreds.* Thus, in 245, the 2 denotes hundreds; the 4, tens; and the 5, units.

3. How many hundreds, how many tens, and how many units in 426? 738? 406? 560? 700? 756? 608? 880? 985? 907? 819? 750? 849?

In reading a number expressed by three figures, the hundreds, tens, and units are read together as so many units. Thus, 245 is read *two hundred and forty-five*, or, more briefly, *two hundred forty-five.*

4. Read 424; 740; 706; 804; 500; 660; 999; 808.

5. Read 90; 89; 460; 606; 781; 808; 990; 100; 250.

TO TEACHERS.—It is increasingly the custom not to use "and" between *integral* orders or periods. For the use of "and" between integral and decimal orders, see page 119.

WRITTEN EXERCISES.

6. Write in words: 300; 350; 465; 506; 708; 750.

Express in figures the following numbers:—

7. Two hundred.
Five hundred.
Seven hundred.
Three hundred forty.
Six hundred seventy.
Nine hundred thirty.

8. Four hundred and five.
Five hundred and six.
Six hundred and four.
Four hundred forty-five.
Eight hundred thirty-seven.
Nine hundred twenty-seven.

9. Express in figures the numbers composed of three hundreds, five tens, and four units; six hundreds, four tens, and three units; five hundreds and seven tens.

10. Express in figures the numbers composed of eight hundreds and six tens; five hundreds and four tens; seven hundreds and five units.

11. What number is composed of 3 hundreds, 0 tens, and 6 units? 2 hundreds and 3 tens? 4 hundreds and 6 units? 5 hundreds and 8 tens?

TWO PERIODS—UNITS AND THOUSANDS.

ORAL EXERCISES.

When a number is expressed by four figures, the fourth or left-hand figure denotes *thousands*. In 4235, the 4 denotes thousands.

1. How many thousands in 4,000? 6,000? 5,400?

Read the following numbers:—

2.	**3.**	**4.**	**5.**	**6.**
1,000	2,200	1,020	2,007	3,432
3,000	4,400	3,040	4,001	4,568
7,000	8,800	7,090	6,005	7,893
9,000	9,900	9,070	8,009	9,890

When a number is expressed by five figures, the fifth or left-hand figure denotes tens of thousands, or *ten-thousands.* Thus, in 45,670, the 4 denotes ten-thousands, and the 5 denotes thousands.

7. How many ten-thousands and how many thousands in 48,500? 85,350? 50,480? 60,070? 92,300?

In reading a number expressed by five figures, the fifth and fourth figures are read together as so many thousands. Thus, 45,400 is read *forty-five thousand four hundred.*

Read the following numbers:—

8.	**9.**	**10.**	**11.**
30,000	44,000	53,440	16,089
50,000	63,000	67,444	99,008
70,000	84,000	48,307	28,045
90,000	99,000	39,600	67,909

When a number is expressed by six figures, the sixth or left-hand figure denotes hundreds of thousands, or *hundred-thousands.* Thus, in 534,000, the 5 denotes hundred-thousands.

12. How many hundred-thousands, how many ten-thousands, and how many thousands in 328,000? 650,000? 508,080? 607,800? 350,307? 470,386?

In reading a number expressed by six figures, the sixth, fifth, and fourth figures are read together as thousands. Thus, 452,500 is read *four hundred fifty-two thousand five hundred.*

Read the following numbers:—

13.	**14.**	**15.**	**16.**
200,000	250,000	845,630	603,408
400,000	360,000	803,084	490,732
600,000	580,000	760,432	308,400
800,000	730,000	900,425	600,550
900,000	960,000	807,708	707,007

The first group of three figures, viz., units, tens, and hundreds, constitutes the first or *units' period.*

In reading numbers the name of the units' period is usually omitted. Thus, 350 is read *three hundred fifty.*

The second group of three figures, viz., thousands, ten-thousands, and hundred-thousands, constitutes the second or *thousands' period.*

The two periods may be separated by a comma, as in the examples above.

17. Read the thousands' period in the 13th, 14th, 15th, and 16th examples above.

18. Read the units' period in the 10th, 11th, 15th, and 16th examples.

WRITTEN EXERCISES.

19. Write in words: 3000; 4060; 7086; 6066; 8006.
20. Write in words: 20000; 25000; 40500; 36300.
21. Write in words: 300000; 440000; 506000; 308400.

Express in figures as one number:—

22. 87 thousand 327 units.
23. 70 thousand 75 units.
24. 200 thousand 2 units.
25. 8 thousand 80 units.
26. 250 thousand 50 units.
27. 305 thousand 305 units.

Express the following numbers in figures:—

28. Three thousand four hundred.
29. Seven thousand six hundred fifty.
30. Four thousand five hundred and six.
31. Forty-five thousand five hundred.
32. Sixty thousand seven hundred ninety.
33. Thirty-eight thousand twenty-two.
34. Sixteen thousand ninety-eight.

35. Sixty-two thousand six hundred.
36. Seventy-three thousand thirty-four.
37. Four hundred fifty thousand two hundred.
38. Seven hundred sixty thousand seven hundred six.
39. Seven hundred eight thousand two hundred.
40. One hundred thousand one hundred one.

THREE PERIODS—UNITS, THOUSANDS, AND MILLIONS.

ORAL EXERCISES.

When a number is expressed by seven or more figures, the seventh figure denotes *millions;* the eighth figure, *ten-millions;* and the ninth figure, *hundred-millions.*

In reading a number expressed by eight figures, the eighth and seventh figures are read together as millions. Thus, 45,050,000 is read *forty-five million fifty thousand.*

Read the following numbers:—

1.	**2.**	**3.**	**4.**
30,000,000	37,300,450	75,000,740	90,090,090
45,000,000	40,207,006	80,000,008	83,830,083
7,000,000	50,350,042	71,017,049	46,060,706
27,000,000	8,080,080	65,560,000	66,666,666

In reading a number expressed by nine figures, the ninth, eighth, and seventh figures are read together as millions. Thus, 450,050,500 is read *four hundred fifty million fifty thousand five hundred.*

Read the following numbers:—

5.	**6.**	**7.**	**8.**
200,000,000	250,200,200	609,070,700	6,006,006
500,000,000	405,360,000	137,030,304	45,045,045
700,000,000	546,308,400	906,089,080	60,060,060
900,000,000	730,070,007	500,500,500	320,320,320

WRITTEN EXERCISES.

9. Write in words: 4,400,300; 40,050,300.
10. Write in words: 65,065,020; 300,070,600.

Write the following numbers in figures:—

11. Fifty million thirty-two thousand six hundred.
12. Thirty-five million nine thousand fifty-six.
13. Five million five thousand five hundred.
14. One million one hundred thousand one hundred.
15. Three hundred million seven hundred thirty.
16. Sixty-two million three hundred thousand.
17. Forty million forty thousand forty.
18. Eighty-six million ten thousand fifty-six.

In the foregoing exercises numbers are expressed by *words* and by *figures.*

The expressing of numbers by figures is *notation*, and the expressing of numbers by words is *numeration.*

There are ten characters called *figures;* viz., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The first (0) is called *naught*, or *cipher.* It is used to fill vacant orders.

There are three orders in each period. In 425, the figure 5 is in the *first* order; the figure 2, in the *second* order; and the figure 4, in the *third* order.

The first period is called *units' period;* the second, *thousands' period;* and the third, *millions' period.*

Ten units equal one ten, ten tens equal one hundred, and ten hundreds equal one thousand; and, generally, *ten ones of any order equal one of the next higher order.*

To Teachers.—This principle of the decimal system can be made plain, if necessary, by the use of the numeral frame, match-sticks, or marks on the board. Care should be taken *not to make the explanation more difficult for pupils than the principle.*

ROMAN NOTATION.

In the Roman Notation, numbers are expressed by means of seven *capital letters;* viz., I, V, X, L, C, D, M.

I represents one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

All other numbers are expressed by repeating or combining these letters.

When a letter is repeated, its value is repeated; thus, II represents 2; XX, 20; CCC, 300, etc.

When a letter is placed before one of greater value, the less value is taken from the greater; thus, IV represents 4; IX, 9; XC, 90.

When a letter is placed after one of greater value, the less value is added to the greater; thus, VI represents 6; XI, 11; CX, 110.

In the following table, numbers are expressed by letters and by figures: —

I,	1	VIII,	8	XV,	15	XL,	40
II,	2	IX,	9	XVI,	16	L,	50
III,	3	X,	10	XVII,	17	LX,	60
IV,	4	XI,	11	XVIII,	18	LXX,	70
V,	5	XII,	12	XIX,	19	LXXX,	80
VI,	6	XIII,	13	XX,	20	XC,	90
VII,	7	XIV,	14	XXX,	30	C,	100

Read the following numbers: —

1.	2.	3.	4.
XIV	CXI	CCL	MDCL
XXIV	CIX	DCXC	MDLX
XXXIV	CLVI	CCXC	MDLIX
XLV	CXLV	DCCL	MDCCC
XCI	CXCV	DCLIX	MDCCCXC
LXIX	CLXV	MCCL	MDCCCXCII

QUESTIONS FOR REVIEW.

What is notation? What is numeration? How many different figures are used to express numbers? Give their names. For what is the cipher used?

How many orders make a period? What is the name of the first period? The second period? The third period?

What orders constitute units' period? Thousands' period? Millions' period?

Which figure in 40642 denotes units? Which denotes tens? Which denotes hundreds? Which denotes ten-thousands?

How many ones of any order make one of the next higher order?

How are numbers expressed in the Roman notation? Name the letters used, and give the value of each.

ADDITION.

ORAL EXERCISES.

1. How many are 11 and 9? 11 and 10? 11 and 12? 11 and 14? 11 and 16? 11 and 18?

2. How many are 12 and 10? 12 and 12? 12 and 13? 12 and 16? 12 and 18? 12 and 15?

3. How many are 13 and 13? 13 and 12? 13 and 15? 13 and 14? 13 and 16? 13 and 17?

4. How many are 14 and 14? 14 and 10? 14 and 12? 14 and 11? 14 and 16? 14 and 15?

5. How many are 15 and 15? 15 and 10? 15 and 11? 15 and 13? 15 and 12? 15 and 14?

6. How many are 16 and 16? 17 and 17? 18 and 18? 19 and 19? 20 and 20? 25 and 25?

The result obtained by adding two or more numbers is their *sum*. The sum of 9 and 7 is 16.

7. What is the sum of 16 and 10? 16 and 16? 18 and 10? 18 and 18? 20 and 20? 25 and 25?

8. What is the sum of 35 and 20? 30 and 25? 40 and 45? 40 and 50? 30 and 50? 50 and 50?

9. What is the sum of 64+8+7+6? Of 37+9+6+8+7? Of 56+9+7+6+8+10+5+8+9?

10. Add by 3's from 2 to 50; by 4's from 24 to 72.

11. Add by 5's from 3 to 53; by 6's from 37 to 83.

12. Add by 7's from 5 to 54; by 8's from 54 to 94.

13. Add by 9's from 4 to 49; by 10's from 45 to 95.

WRITTEN EXERCISES.

14. What is the sum of 121, 233, 123, 332, 231, 323, 123, 232, and 333?

Write the numbers so that the units shall form the first column; the tens, the second column; and the hundreds, the third column.

Begin with the units' column, and add, naming results only; thus, 5, 8, 11, 12, 14, 17, 20, 21,—21 units, equal 2 tens and 1 unit.

Write the 1 unit under the units' column, and add the 2 tens with the tens' column; thus, 5, 8, 10, 12, 15, 18, 20, 23, 25,—25 tens, equal 2 hundreds and 5 tens.

Write the 5 tens under the tens' column, and add the 2 hundreds with the hundreds' column; thus, 5, 7, 8, 11, 13, 16, 17, 19, 20,—20 hundreds, equal 2 thousands and 0 hundreds.

Write the 0 hundreds under the hundreds' column, and write the 2 thousands in thousands' place. The sum is 2051.

PROCESS.

$$\begin{array}{rl}
121 & \\
233 & \\
123 & \\
332 & \\
231 & \\
323 & \\
123 & \\
232 & \\
\underline{333} & \\
\underline{\underline{2051}}, & \textit{Sum.} \\
21 & \\
23 & \\
\underline{18} & \\
2051, & \textit{Sum.}
\end{array}$$

To test the accuracy of the work, add the columns downward.

The manner in which the sums of the several columns are united is shown below the double lines.

To **Teachers.**—For blackboard drills in addition see *Oral Lessons in Number*, pages 155–158.

15. Add 256, 308, 5763, 48065, 9639, 4877, and 66785.

16. Add 16405, 9807, 84063, 46088, 709, 7008, 76509.

Copy and add the following numbers:—

17.	**18.**	**19.**	**20.**	**21.**
33232	15215	23512	52134	24137
32323	14343	30425	34445	16126
23213	45046	41341	53054	20050
13221	50350	23301	44052	16654
32233	33432	41545	25253	33456
32111	43543	43453	34545	44162
23212	23343	25445	41534	23206
32021	45452	41505	22335	36562

22.	**23.**	**24.**	**25.**	**26.**
32620	19864	7121365	42764	5784062
75437	34687	2171634	8738768	309984
50743	46768	1237773	634187	67792
64017	65837	7143656	9463506	9999
32516	80040	2674467	324483	999807
18416	18608	6734765	43832	9095742
13673	36084	6574636	2741608	3407856
31654	45687	7147347	837860	9900789
46810	38762	8342108	708078	444444
53472	77777	3562735	666666	666666
16172	88888	4444444	555555	99999

27.	**28.**	**29.**	**30.**
$271.50	$ 281.65	$40091.00	$7990.38
452.63	414.12	2700.36	6911.04
580.16	1755.50	1160.23	1365.85
121.41	2506.00	10850.75	1177.60
957.00	1827.37	20.63	858.91
8.55	4570.19	471.84	5708.97
30.38	61.18	4907.37	4641.09
630.25	30.27	19278.08	1550.25

NOTE.—In adding sums of money, the dollar sign is written but twice; viz, before the first number and before the answer.

31. Add $131.45, $91.87, $100.50, $10.87, $960.50, $751.87, $554.75, $2500.84, and $160.25.

32. Add $2.60, $3.72, $136.87, $458.60, $450.83, $908.50, $112.83, $59.84, $60.75, and $18.05.

33. What is the sum of four hundred four; four thousand forty; forty thousand four hundred; and four million four hundred thousand?

34. What is the sum of thirty-six thousand three hundred twenty-five; fourteen thousand forty-six; twenty-three thousand four hundred five; fifteen thousand sixteen; and three hundred thousand three hundred four?

35. What is the sum of three million one thousand fifty-six; six hundred thousand six hundred twenty-five; four million forty-two thousand four; and forty-five million six hundred fifty thousand?

36. What is the sum of sixteen million four thousand sixty-five; three hundred thousand two hundred fifty-six; seven thousand forty; five million five thousand seven; four hundred thousand six hundred seven?

37. What is the sum of forty-five million seven thousand seventy; six million sixty-five thousand two hundred six; seventy-five thousand forty-four; eight million eight thousand eight; eighteen million eighteen thousand eighteen; and eighty million eight hundred thousand?

How is the sum of two or more numbers found? How many ones in the sum?

How are the numbers to be added written? At which order do you begin to add?

How is the sum of the units' column written when it is less than ten? When ten or more?

How is the sum of any column written when it is less than ten? When ten or more?

How is the sum of the left-hand column written when it is ten or more?

How may the accuracy of the result be tested?

PROBLEMS IN ADDITION.

ORAL PROBLEMS.

1. Frank has 5 marbles in one hand and 2 marbles in the other: how many has he in both hands?

SOLUTION.—5 marbles and 2 marbles are 7 marbles: Frank has 7 marbles in both hands.

2. A drover bought 9 sheep of one farmer and 7 sheep of another: how many sheep did he buy?

3. A grocer sold 8 pounds of sugar to one customer, 3 pounds to another, and 2 pounds to another: how many pounds of sugar did he sell?

4. A man walked 4 miles the first hour, 3 miles the second, and 2 miles the third: how many miles did he walk in the 3 hours?

5. A man gave $26 for a coat and $5 for a hat: how many dollars did he give for both?

6. A drover bought 19 cows of one man, 4 cows of another, and 5 of another: how many cows did he buy?

7. James picked 27 peaches from one limb and 5 peaches from another: how many peaches did he pick from both limbs?

8. Mary has written 16 lines: if she write 5 lines more, how many lines will she then have written?

9. George gave 15 cents for a slate and 5 cents for a pencil: how many cents did he give for both?

10. John solved 18 problems before school and 6 problems in school: how many problems did he solve?

11. A farmer bought a cow for $27 and a calf for $6: how much did he pay for both?

12. The head of a fish is 5 inches long, its body 16 inches, and its tail 6 inches: how long is the fish?

13. In an orchard there are 29 apple-trees, 5 pear-trees, and 6 peach-trees: how many trees in the orchard?

14. William gave a blind boy 19 cents, John gave him 15 cents, and Charles gave him 6 cents: how many cents did they all give him?

15. A man bought a harness for $37, a saddle for $7, and a bridle for $4: how much did he pay for all?

16. Frank gave 10 cents for a lead-pencil, 5 cents for a piece of rubber, and 7 cents for paper: how much did the three articles cost?

17. A gentleman gave $36 for a suit of clothes, $7 for a pair of boots, and $5 for a hat: how much did he pay for all?

18. A farmer raised 16 loads of hay in one field, 8 loads in another, and 7 loads in another: how many loads of hay did he raise?

19. A lady paid $27 for a shawl, $8 for a hat, and $4 for shoes: how much did she pay for all?

20. A merchant sold 18 yards of muslin to one customer, 7 yards to another, and 8 yards to another: how many yards did he sell?

21. A man paid $23 for a coat, $9 for a pair of trousers, and $6 for a vest: what was the cost of the suit?

22. A lady gave 15 cents for thread, 8 cents for needles, and 7 cents for pins: how much did she spend?

23. A gentleman gave $95 for a horse, $15 for a saddle, and $5 for a bridle: how much did he pay for all?

WRITTEN PROBLEMS.

24. January has 31 days; February, (except in leap year,) 28; March, 31; April, 30; May, 31; and June, 30: how many days in the first six months of the year?

25. July has 31 days; August, 31; September, 30; October, 31; November, 30; and December, 31: how many days in the last six months of the year?

26. A man bought four loads of hay, the first weighing 2130 pounds, the second 2312 pounds, the third 2232 pounds, and the fourth 2322 pounds: how many pounds of hay in the four loads?

27. A grain-dealer bought 2350 bushels of wheat on Monday, 4215 bushels on Tuesday, 3245 bushels on Wednesday, 1500 bushels on Thursday, 1424 bushels on Friday, and 1350 bushels on Saturday: how many bushels did he buy?

28. In a city having five wards, there are 345 voters in the first ward, 443 in the second, 213 in the third, 523 in the fourth, and 425 in the fifth: how many voters in the city?

29. The first ward of a city contains 1675 youth of school-age; the second, 2357 youth of school-age; the third, 2347; the fourth, 3270; and the fifth, 2677: how many youth of school-age in the city?

30. A father gave to his eldest son 225 acres of land; to the second, 155 acres; to the third, 145 acres; and to the youngest, 124 acres: how many acres did he give to all?

31. The first three cars of a freight train contain 25240 pounds each; the next four cars, 25345 pounds each; the next two cars, 31540 pounds each; and the last car, 25432 pounds: how much freight in the ten cars?

32. A man paid $3575 for a lot, $5460 for a house, $875 for a stable, and $675 for other improvements: what did the property cost?

33. A man owns 5 farms, containing respectively 285 acres, 345 acres, 146 acres, 438 acres, and 248 acres: how many acres of land does he own?

34. A steam-ship sailed 217 miles the first day; 265 miles the second; 227 miles the third; 187 miles the fourth; and 168 miles the fifth: how many miles did it sail in the five days?

35. The distance by railroad from Boston to Springfield is 98 miles; from Springfield to Albany, 102 miles; from

Albany to Buffalo, 296 miles; from Buffalo to Cleveland, 183 miles; from Cleveland to Chicago, 357 miles: how far from Boston to Chicago?

36. A merchant's sales on Monday were $275; on Tuesday, $368; on Wednesday, $540; on Thursday, $165; on Friday, $398; and on Saturday, $754: what was the amount of his sales in the week?

37. Ohio contains 41060 square miles; Indiana, 36350 square miles; Illinois, 56650 square miles; and Michigan, 58915 square miles: how many square miles in these four States?

38. Maine contains 33040 square miles; New Hampshire, 9305; Vermont, 9565; Massachusetts, 8315; Rhode Island, 1250; and Connecticut, 4990: how many square miles in these six New England States?

39. A merchant deposited in bank on successive days $345, $564, $849, $788, $1285, and $2684: how much money did he deposit?

Addition is the process of finding the sum of two or more numbers. The sum contains as many ones as all the numbers added.

Nine balls and 12 balls are numbers of the *same kind*, and 6 tens and 3 tens are numbers of the *same order*.

Numbers of the same kind or order are *like numbers*.

Only like numbers can be added.

Three pears and 4 chairs can not be added, and 3 tens and 4 units can only be added by first reducing the 3 tens to 30 units, thus making the two numbers to be added *like* numbers. This shows why in addition units of the same order must be written in the same column.

A *concrete* number is a number applied to a particular thing; as, 4 pears, 15 steps, 9 hours, $16, etc.

An *abstract* number is a number not applied to a particular thing; as, 5, 13, 25, 100, etc.

SUBTRACTION.

ORAL EXERCISES.

1. How many is 11 less 3? 21 less 3? 31 less 3? 12 less 3? 22 less 3? 32 less 3?

2. How many is 13 less 4? 23 less 4? 43 less 4? 13 less 5? 33 less 5? 53 less 5? 63 less 5?

3. How many is 13 less 6? 33 less 6? 63 less 6? 14 less 6? 44 less 6? 54 less 6?

4. How many is 12 less 7? 32 less 7? 52 less 7? 62 less 7? 15 less 7? 45 less 7? 55 less 7?

5. How many is 15 less 8? 25 less 8? 45 less 8? 35 less 8? 65 less 8? 75 less 8? 95 less 8?

6. How many is 17 less 8? 37 less 8? 57 less 8? 47 less 8? 67 less 8? 77 less 8? 97 less 8?

7. How many is 16 less 9? 36 less 9? 46 less 9? 26 less 9? 66 less 9? 86 less 9? 76 less 9?

8. How many is 13 less 9? 33 less 9? 43 less 9? 63 less 9? 53 less 9? 73 less 9? 83 less 9?

9. How many is 23 less 11? 24 less 12? 25 less 12?

10. How many is 25 less 14? 30 less 15? 42 less 20?

11. Subtract by 2's from 50 back to 20; from 41 to 21.

12. Subtract by 6's from 36 back to 0; from 48 to 30.

13. Subtract by 5's from 55 back to 30; from 30 to 0.

14. Subtract by 8's from 64 back to 32; from 32 to 0.

The result found by taking one number from another is their *difference.*

The greater number is the *minuend;* the less number is the *subtrahend.*

15. From 26 cents take 12 cents. Which number is the minuend? The subtrahend? The difference?

16. The minuend is 18 and the subtrahend 12: what is the difference?

WRITTEN EXERCISES.

17. From 358 take 235; from 3683 take 2542.

18. From 5334 take 2726.

PROCESS.
5334
2726
2608, *Difference.*

Write units under units, tens under tens, etc.

Since 6 units can not be taken from 4 units, add 10 units to the 4 units, making 14 units, and take 6 units from the 14 units, and write 8 units (the difference) below. Add 1 ten (equal to 10 *units*) to the 2 tens, making 3 tens, and take 3 tens from 3 tens, and write 0 (the difference) below.

Since 7 hundreds can not be taken from 3 hundreds, add 10 hundreds to the 3 hundreds, making 13 hundreds, and take 7 hundreds from 13 hundreds, and write 6 hundreds (the difference) below. Add 1 thousand (equal to 10 *hundreds*) to the 2 thousands, making 3 thousands, and take 3 thousands from 5 thousands, and write 2 thousands (the difference) below. The difference is 2608.

To Teachers.—The adding of 10 to a term of the minuend and 1 to the next higher term of the subtrahend *increases both minuend and subtrahend equally*, and hence does not affect the difference.

The 10 added is not "borrowed," and no reductions of the higher terms of the minuend are necessary. The pupil simply adds 10 to the minuend and 1 (equal to the 10) to the subtrahend.

Instead of adding 1 to the next higher term of the subtrahend, 1 may be taken from the next higher term of the minuend. Many teachers prefer this method. There is no material difference between the two methods. In first taking 1 from the minuend term and then subtracting the subtrahend term, there are *two* subtractions. If the 1 be added to the subtrahend term, there is but *one* subtraction. See *Oral Lessons in Number*, page 175.

	19.	**20.**	**21.**	**22.**	**23.**
From	463	3272	1385	5754	3416
Take	336	2147	1276	3457	507

	24.	**25.**	**26.**	**27.**	**28.**
From	$30.41	$30.75	$705.63	$500.00	$304.06
Take	26.37	18.47	294.87	90.45	70.85

29. From sixteen thousand twenty-six take seven thousand five hundred forty-five.

30. From five hundred seven thousand two hundred two take eighty-seven thousand sixty-seven.

31. From forty million seven hundred take eighty thousand eighty-eight.

32. From fifty-seven million six thousand sixty take eight million eight thousand eighty-four.

How is the difference between two numbers found? Which number is the minuend? Which is the subtrahend?

In subtraction how are the numbers written?

At which order do you begin to subtract? Where do you write the difference?

When a lower number of any order is greater than the upper number, how do you proceed?

How may the accuracy of the result be tested?

PROBLEMS IN SUBTRACTION.

ORAL PROBLEMS.

1. Charles earned 21 cents by selling papers, and gave 4 cents for a comb: how many cents had he left?

Solution.—21 cents less 4 cents are 17 cents; Charles had 17 cents left.

2. Kate is 15 years old and her sister is 6 years younger: what is her sister's age?

3. There are 21 passengers in a car: if 7 of them leave at a station, how many will remain?

4. There are 13 men in one coach and 8 men in another: how many men in the first coach more than in the second?

5. A man gave $12 for a saddle and $4 for a bridle: how much did the saddle cost more than the bridle?

6. Henry wrote 21 words, but misspelled 5 of them: how many words did he spell correctly?

7. Charles's lesson consists of 15 problems, and he has solved 9 of them: how many has he not solved?

8. A school has enrolled 65 pupils, and 8 pupils are absent: how many are present?

9. In a school, 63 pupils are enrolled and 54 are present: how many pupils are absent?

10. From a cask containing 45 gallons of molasses, 39 gallons were sold: how many gallons remained unsold?

11. A boy earned 50 cents by selling papers, and then paid 25 cents for a reader, 8 cents for a slate, and 10 cents for a copy-book: how many cents had he left?

12. A man earns $45 a month; if he pay $10 per month for house rent, $8 for meat, and $12 for groceries, how much will he have left?

WRITTEN PROBLEMS.

13. A grocer bought 585 pounds of sugar and sold 231 pounds: how many pounds had he left?

14. A farmer raised 1560 bushels of wheat and sold 875 bushels: how many bushels were left?

15. A man bought a house for $4370, and sold it for $6450: how much did he gain?

16. A man bought 3487 bushels of wheat, and sold 1495 bushels: how many bushels had he left?

17. The number of youth of school-age in a certain city is 1234, and 756 pupils are enrolled in the schools: how many youth of school-age are not enrolled?

18. The population of the United States in 1870 was 38558371, and in 1880 it was 50155783: what was the increase in ten years?

19. A merchant having $11315 in bank, drew out $976: how much remained in the bank?

20. The Pilgrims landed at Plymouth in 1620, and our National Independence was declared in 1776: how many years between the two events?

21. The area of the United States is 3602990 square miles, and the area of Europe is 3928252 square miles: how much larger is Europe than the United States?

22. The population of the United States in 1880 was 43475840 native, and 6679943 foreign born: how many native persons more than foreign born?

23. America was discovered in 1492, and the Pilgrims landed at Plymouth in 1620: how many years intervened?

24. The first steam-boat was made in 1807, and the Atlantic Cable was laid in 1866: how many years between the two events?

25. Mt. Etna is 10874 feet high, and Mt. Vesuvius 3948 feet: how much higher is Etna than Vesuvius?

26. Mont Blanc, in Europe, is 15744 feet high, and Mount Sorata, in South America, is 21286 feet high: what is the difference in their heights?

27. The six New England States contain 66465 square miles, and California contains 158360 square miles: how much greater in area is California than New England?

Subtraction is the process of taking a less number from a greater number. The result is the *difference.*

The sum of the difference and the less number equals the greater number.

Only like numbers can be subtracted.

Three pencils can not be taken from 4 books, and 3 units can be taken from 4 tens only by first reducing the tens to units, thus making minuend and subtrahend *like* numbers. In subtraction, units must be written under units, tens under tens, hundreds under hundreds, etc.

PROBLEMS COMBINING ADDITION AND SUBTRACTION.

1. Robert picked 21 peaches, and gave 7 to his sister and 8 to his brother: how many peaches had he left?

SOLUTION.—7 peaches and 8 peaches are 15 peaches; 21 peaches less 15 peaches are 6 peaches: Robert had 6 peaches left.

2. A man earned $45, and paid $15 for house rent, $8 for flour, $7 for shoes, and $10 for groceries: how much had he left?

3. There are 15 pigeons on one branch of a tree, and 9 pigeons on another branch: if 7 should fly away, how many would be left on the tree?

4. A drover bought 17 sheep of one farmer, 6 sheep of another, and 8 of another, and then sold 9 of them to a butcher: how many sheep had he left?

5. Harry bought 2 dozen marbles, and gave 8 marbles to his brother and 6 to his sister: how many had he left?

WRITTEN PROBLEMS.

6. From the sum of $4750 and $2284, take $5863.

7. From the sum of $3040, $2685, and $872, take the sum of $5231 and $165.

8. A man deposited $175, $141, $75, $304, and $250 in a bank, and then drew out $480 and $225: how much did he deposit? How much did he draw out? How much remained in the bank?

9. The area of North America is 9349585 square miles, and of South America, 6889135 square miles: what is the area of the Western continent?

10. The area of Asia is 17117204 square miles; of Africa, 11511500; and of Europe, 3928252: what is the area of the Eastern continent? How much greater is the Eastern continent than the Western?

MULTIPLICATION.

ORAL EXERCISES.

1. How many are 4 times 3? 6 times 3? 8 times 3? 10 times 3? 5 times 3? 7 times 3? 9 times 3?

2. How many are 4 times 4? 6 times 4? 8 times 4? 10 times 4? 5 times 4? 7 times 4? 9 times 4?

3. How many are 3 times 5? 5 times 5? 7 times 5? 9 times 5? 6 times 5? 8 times 5? 10 times 5?

4. How many are 5 times 6? 7 times 6? 9 times 6? 4 times 6? 6 times 6? 8 times 6? 10 times 6?

5. How many are 3 times 7? 5 times 7? 7 times 7? 9 times 7? 4 times 7? 6 times 7? 8 times 7?

6. How many are 4 times 8? 6 times 8? 8 times 8? 10 times 8? 5 times 8? 7 times 8? 9 times 8?

7. Multiply 5 by 6; 15 by 6; 25 by 6; 25 by 8.

8. Multiply 12 by 8; 12 by 10; 12 by 12.

The result obtained by multiplying one number by another is their *product.*

The number multiplied is the *multiplicand,* and the number by which it is multiplied is the *multiplier.*

9. Multiply 25 by 6: what is the product? Which number is the multiplicand? Which is the multiplier?

10. If 15 be multiplied by 20, what will be the product? Which of the numbers is the multiplicand? Which is the multiplier?

WRITTEN EXERCISES.

11. Multiply 345 by 6; by 8; by 7.

12. Multiply 563 by 5; by 7; by 8; by 6.

13. Multiply 5674 by 4; by 7; by 9; by 8.

14. Multiply 6708 by 6; by 5; by 7; by 9; by 8.

15. Multiply 8099 by 5; by 6; by 7; by 8; by 9.
16. Multiply 2706 by 6, and the product by 8.
17. Multiply 4560 by 7, and the product by 9.
18. Multiply 3048 by 8, and the product by 7.
19. Multiply 456 by 43.

PROCESS.

$$\begin{array}{lr} \textit{Multiplicand,} & 456 \\ \textit{Multiplier,} & 43 \\ \hline & 1368 \\ & 1824 \\ \hline \textit{Product,} & 19608 \end{array}$$

ILLUSTRATIVE PROCESS.

$$\begin{array}{lr} 456 \times 3 = & 1368 \\ 456 \times 40 = & 18240 \\ \hline \textit{Product,} & 19608 \end{array}$$

Write the multiplier under the multiplicand, placing units under units and tens under tens.

First multiply by the 3 units (as in the preceding examples), which gives 1368 for the *first* partial product.

Next multiply by the 4 tens, and since units multiplied by tens (or tens by units) produce *tens*, the first figure of the second partial product will denote *tens*. Write the 4 tens under tens, and the other figures to the left in order.

Add the two partial products, and their sum, which is 19608, is the product required.

To TEACHERS.—Show that units multiplied by tens produce *tens*; tens by tens, *hundreds*; hundreds by tens, *thousands*, etc.; and hence that the first figure of each partial product is written under the multiplier which produces it.

20. Multiply 4606 by 54; by 75; by 86; by 69.
21. Multiply 2327 by 36; by 65; by 78; by 84.
22. Multiply 30279 by 34; by 57; by 66; by 88.
23. Multiply 43065 by 41; by 61; by 50; by 80.
24. Multiply 34756 by 65; by 71; by 60; by 90.
25. Multiply 40732 by 55; by 66; by 88; by 77.
26. Multiply 4068 by 346.
27. Multiply 63082 by 435.
28. Multiply 40694 by 327.
29. Multiply 45053 by 644.
30. Multiply 343607 by 482.
31. Multiply 60563 by 2346.
32. Multiply 5378 by 2435.
33. Multiply 3254 by 425.

(26) PROCESS.

$$\begin{array}{rl} 4068 & \\ 346 & \\ \hline 24408 & \\ 16272 & \\ 12204 & \\ \hline 1407528, & \textit{Product.} \end{array}$$

34. Multiply 2346 by 2346.
35. Multiply 4165 by 4165; by 561.
36. Multiply 256 by 256, and the product by 35.
37. Multiply 325 by 325, and the product by 325.
38. Multiply 342 by 342, and the product by 342.
39. Multiply 453 by 453, and the product by 453.
40. Multiply 2745 by 306.
41. Multiply 4086 by 608.
42. Multiply 7908 by 506.
43. Multiply 6075 by 3008.
44. Multiply 3460 by 4008.
45. Multiply 4607 by 5006.

(40) PROCESS.
2745
306
16470
8235
839970, *Product.*

46. Multiply 5083 by 706; by 907; by 6005.
47. Multiply 45063 by 706; by 7008; by 6307.
48. Multiply 30876 by 705; by 3406; by 7009.
49. Multiply 14800 by 47; by 504; by 6065.
50. Multiply 348000 by 324; by 462; by 503.
51. Multiply 230800 by 405; by 666.
52. Multiply 23050 by 4006; by 6005.
53. Multiply 2059 by 3400.
54. Multiply 4306 by 67000.
55. Multiply 5284 by 2900.
56. Multiply 4509 by 56000.
57. Multiply 6375 by 40800.
58. Multiply 92005 by 6800.

(53) PROCESS.
2059
3400
8236
6177
7000600, *Product.*

59. Multiply 89000 by 4500; by 5600.
60. Multiply 94000 by 3600; by 4300.
61. Multiply 90800 by 640; by 1200.
62. Multiply 4760 by 203000; by 45000.
63. Multiply 88000 by 8800; by 4040.
64. Multiply 3074 by 3400; by 4000.
65. Multiply 30840 by 5000; by 840.
66. Multiply $36580 by 425; by 708; by 240.
67. Multiply $4016 by 24; by 90; by 205; by 700.
68. Multiply $74 by 100; by 1000; by 10000.
69. Multiply $306 by 100; by 10000; by 1000.

How is the product of two numbers found? Which number is the multiplicand? Which is the multiplier?

How are the numbers to be multiplied written? How do you multiply when the multiplier is less than ten?

How do you multiply when the multiplier has more than one term? Where is the first figure of each partial product written?

What is done with the several partial products to find the true product?

How do you multiply when the multiplier ends with one or more ciphers?

PROBLEMS IN MULTIPLICATION.

ORAL PROBLEMS.

1. If a man earn $3 a day, how many dollars will he earn in 6 days?

SOLUTION.—If a man earn $3 in one day, in 6 days he will earn 6 times $3, which is $18.

2. If a boy walk 3 miles a day in attending school, how many miles will he walk in 10 days?
3. There are 3 feet in a yard: how many feet in 2 yards? In 4 yards? 5 yards? 7 yards?
4. There are 4 quarts in a gallon: how many quarts in 5 gallons? In 7 gallons? 10 gallons?
5. There are 5 cents in a half-dime: how many cents in 3 half-dimes? In 5 half-dimes?
6. If there are 5 school-days in a week, how many school-days in 6 weeks? In 8 weeks? 12 weeks?
7. If a horse travel 6 miles an hour, how far will it travel in 5 hours? In 8 hours? In 10 hours?
8. There are 6 feet in a fathom: how many feet in 7 fathoms? In 9 fathoms? 8 fathoms?
9. There are 6 days for labor in each week: how many days for labor in 6 weeks? In 9 weeks? 10 weeks?
10. There are 8 rows of trees in an orchard, and 6 trees in each row: how many trees in the orchard?

11. There are 4 pecks in a bushel: how many pecks in 6 bushels? In 8 bushels? 10 bushels? 12 bushels?

12. There are 8 quarts in 1 peck: how many quarts in 3 pecks? In 5 pecks? 7 pecks? 6 pecks?

13. There are 8 pints in a gallon: how many pints in 4 gallons? In 6 gallons? 8 gallons? 10 gallons?

14. If an orange is worth 5 apples, how many apples are 7 oranges worth? 10 oranges? 8 oranges?

15. James has 8 marbles, and John has 6 times as many: how many marbles has John?

16. If a man earn $8 a week, how much will he earn in 9 weeks? In 11 weeks? 8 weeks? 10 weeks?

17. A railroad car has 8 wheels: how many wheels has a train of 7 cars? A train of 9 cars?

18. If a horse eat 8 quarts of oats each day, how many quarts will he eat in 6 days? In 10 days?

19. Charles earns $9 a month as errand-boy: how much will he earn in 10 months? In 9 months?

20. What will 7 lead-pencils cost at 6 cents apiece?

21. What will 6 oranges cost at 5 cents apiece?

22. If a pint of oil cost 8 cents, what will 8 pints cost? 10 pints? 12 pints? 20 pints?

23. What will 6 bananas cost at 2 for 5 cents?

24. What will 5 barrels of flour cost at $9 a barrel?

25. What will 8 pounds of beef cost at 10 cents a pound? At 12 cents? At 15 cents?

26. What will 6 quarts of strawberries cost at 8 cents a quart? At 10 cents? At 12 cents?

27. What will 12 heads of cabbage cost at 4 cents a head? At 5 cents? At 6 cents? At 10 cents?

28. What will 8 boxes of matches cost at 15 cents a box? At 20 cents? At 25 cents?

29. What will 4 pairs of boots cost at $6 a pair?

30. A man earns $9 a week, and pays $5 a week for board and washing: how much can he save in 1 week? In 8 weeks? 10 weeks? 15 weeks?

31. If a man earn $10 a week and a boy $5 a week, how much will they both earn in 1 week? In 8 weeks?

32. A man bought 8 yards of cloth at 75 cents a yard, and gave in payment a $10 bill: how much money did he receive in return?

WRITTEN PROBLEMS.

33. There are 320 rods in a mile: how many rods are there in 3 miles? In 12 miles? 25 miles?

34. If a train of cars run 425 miles a day, how far will it run in 8 days? In 45 days? 160 days?

35. If 135 tons of iron rails will make one mile of railroad, how many tons will make 245 miles?

36. If a ship sail 216 miles a day, how far will it sail in 12 days? In 23 days? In 45 days?

37. If a steamer sail 20 miles an hour, how far will it sail in 6 days of 24 hours each?

38. A father divided his estate among 4 sons, giving to each $3545: what was the value of the estate?

39. There are 60 minutes in an hour: how many minutes in 24 hours, or one day? How many minutes in 7 days, or a week? In 8 weeks?

40. If Enos lived 905 years, how many days did he live, allowing 365 days to the year?

41. A planter raised 208 bales of cotton, each bale weighing 440 pounds: how many pounds of cotton did he raise?

42. A farmer raised 75 acres of wheat, and it yielded an average of 28 bushels per acre: how many bushels of wheat did he raise?

43. What will it cost to build 305 miles of railroad at $7525 a mile?

44. There are 5280 feet in a mile: how many feet in 80 miles? In 75 miles? 605 miles?

45. The earth moves in its orbit at an average rate of 68400 miles in an hour: how far does it move in 24 hours? In 48 hours? In 120 hours?

46. If a carriage-wheel revolve 280 times in running a mile, how many times will it revolve in running 18 miles? 75 miles? 250 miles?

47. A boat is loaded with 245 bales of hay, weighing 280 pounds each: what is the weight of the cargo?

48. There are 24 sheets of paper in a quire and 20 quires in a ream: how many sheets in a ream? In 80 reams? In 240 reams?

49. There are 12 things in a dozen, and 12 dozens in a gross: how many pencils in one gross? In 65 gross?

50. A steam-boat made 145 trips in a season, and carried, on an average, 280 passengers each trip: how many passengers did she carry during the season?

51. There are 3600 seconds in one hour: how many seconds are there in 24 hours? In 168 hours?

52. Light moves 192000 miles in a second: how far does it move in 60 seconds, or one minute? In one hour? In 24 hours, or one day?

53. Sound moves 1090 feet in a second: how far will it move in 60 seconds, or one minute? In 60 minutes, or an hour? In 24 hours?

54. How far will a railroad train run in 3 days, of 24 hours each, if it run on an average 32 miles an hour?

55. What will 1700 tons of railroad iron cost at $75 a ton? At $84 a ton? At $66 a ton?

56. A grocer bought 75 boxes of soap, each containing 60 pounds, at 5 cents a pound: what was the cost?

57. A farmer sold 90 tons of hay at $12 a ton: how much did he receive?

58. A drover bought 120 head of cattle at an average cost of $37 a head: how much did the cattle cost?

59. What will 640 acres of land cost at $62 an acre? At $25 an acre? At $33 an acre?

60. A merchant bought 5 dozens of silver watches at $14 apiece: how much did they cost?

61. What will 240 barrels of flour cost at $6 a barrel? At $5 a barrel? At $8 a barrel?

62. An army is composed of 54 regiments, each containing, on an average, 670 men: how many men in the army?

63. A grocer bought 68 barrels of sugar, each weighing 275 pounds, at 8 cents a pound: how much did the sugar cost?

64. A salesman receives $90 per month, and his monthly expenses are $45: how much does he save in a year? In 18 years?

65. The President's salary is $50000 a year: if his expenses are $2500 a month how much can he save during his term of 4 years?

66. A man earns $1800 a year and his expenses average $4 a day: how much will he save in 365 days, or one year? How much in 9 years?

67. A drover bought 180 head of cattle in Ohio at $45 a head, shipped them to New-York at an expense of $6 a head, and then sold them at $56 a head: how much did he make?

68. A miller manufactured 560 barrels of flour, and sold it at $9 a barrel; the wheat cost $2750, and the expense of running the mill was $960: how much did he make?

69. A train of 27 cars is loaded with iron; each car contains 96 bars, and each bar weighs 184 pounds: what is the weight of all the iron?

Multiplication is the process of taking one number as many times as there are units in another.

Multiplication may be considered as a short method of finding the sum obtained by adding a number to itself. The product of 5 multiplied by 4 is the sum of 5 + 5 + 5 + 5.

DIVISION.

ORAL EXERCISES.

1. How many 2's in 8? 2's in 18? 2's in 24? 3's in 15? 3's in 21? 3's in 27?

2. How many 4's in 24? 4's in 36? 4's in 48? 5's in 35? 5's in 45? 5's in 50? 5's in 60?

3. How many 6's in 30? 6's in 42? 6's in 54? 7's in 35? 7's in 56? 7's in 42? 7's in 63?

4. How many 8's in 40? 8's in 64? 8's in 72? 9's in 63? 9's in 54? 9's in 81? 9's in 90?

5. How many times is 2 contained in 6? 2 in 10? 2 in 14? 2 in 18? 2 in 16? 2 in 12?

6. How many times is 3 contained in 9? 3 in 15? 3 in 21? 3 in 27? 3 in 24? 3 in 36? 3 in 32?

Suggestion.—3 is contained in 32 ten times, with 2 remainder.

7. How many times 4 in 8? 4 in 24? 4 in 25? 4 in 26? 4 in 40? 4 in 28? 4 in 30? 4 in 45?

8. How many times 5 in 25? 5 in 35? 5 in 45? 5 in 55? 5 in 32? 5 in 42? 5 in 52? 5 in 62?

9. How many times 6 in 54? 6 in 56? 6 in 58? 6 in 62? 6 in 50? 6 in 38? 6 in 48? 6 in 52?

10. How many times 7 in 70? 7 in 77? 7 in 84? 7 in 53? 7 in 73? 7 in 75? 7 in 50? 7 in 53?

11. How many times 8 in 32? 8 in 40? 8 in 64? 8 in 56? 8 in 72? 8 in 88? 8 in 84? 8 in 90?

12. How many times 9 in 45? 9 in 48? 9 in 63? 9 in 68? 9 in 72? 9 in 77? 9 in 95?

13. How many times 10 in 50? 10 in 65? 10 in 96? 10 in 95? 10 in 98? 10 in 110? 10 in 120?

14. What is $\frac{1}{3}$ of 12? $\frac{1}{4}$ of 36? $\frac{1}{5}$ of 35? $\frac{1}{6}$ of 42? $\frac{1}{7}$ of 63? $\frac{1}{8}$ of 72? $\frac{1}{9}$ of 54? $\frac{1}{10}$ of 120?

15. What is $\frac{1}{4}$ of 13? $\frac{1}{5}$ of 23? $\frac{1}{7}$ of 36? $\frac{1}{8}$ of 65? $\frac{1}{9}$ of 65? $\frac{1}{10}$ of 81? $\frac{1}{11}$ of 56? $\frac{1}{12}$ of 60? $\frac{1}{12}$ of 61?

WRITTEN EXERCISES.

16. Divide 80484 by 4; by 3.
17. Divide 808404 by 4; by 6.
18. Divide 18480 by 6; by 8.
19. Divide 87507 by 7; by 9.
20. Divide 648240 by 6; by 8.
21. Divide 398604 by 3; by 6.
22. Divide 10784 by 5; by 9; by 8.

(16) PROCESS.

$$\begin{array}{r} 4\overline{)80484} \\ \hline 20121, \text{ \textit{Quotient.}} \end{array}$$

The above written process is called *short division.*

When there is a remainder after dividing the last term of the dividend, write the remainder below; or write the remainder over the divisor with a line between them and at the right of the quotient. 10784 divided by 5 gives 2156, with 4 as a remainder, or it gives 2156$\frac{4}{5}$. The latter form is preferable.

PROCESS.

$$\begin{array}{r} 5\overline{)10784} \\ \hline 2156, \text{ \textit{Quotient.}} \\ 4, \text{ \textit{Remainder.}} \end{array}$$

$$\begin{array}{r} 5\overline{)10784} \\ \hline 2156\frac{4}{5}, \text{\textit{Quotient.}} \end{array}$$

23. Divide 1606075 by 4; by 6; by 5; by 8.
24. Divide 4706270 by 3; by 4; by 7; by 9.
25. Divide 10632403 by 5; by 6; by 8; by 7.
26. Divide $207841 by 3; by 6; by 8; by 9.
27. Divide $4038475 by 5; by 4; by 6; by 8.
28. Divide 543020 by 5; by 10; by 11; by 12.
29. Divide 1030507 by 3; by 5; by 7; by 9.

The result obtained by dividing one number by another is their *quotient.*

The number which is divided is the *dividend*, and the number by which it is divided is the *divisor.*

The part of the dividend which remains undivided is the *remainder.*

There is no remainder when the dividend contains the divisor an exact number of times.

30. Divide 60703 by 7. What is the quotient? Which number is the dividend? Which is the divisor?

31. Divide 85763 by 8. What number is the quotient? The remainder? The divisor? The dividend?

32. Multiply 248 by 6 and divide the product by 8. What number is the dividend? The divisor?

LONG DIVISION.

1. Divide 60703 by 7 by long division.

2. Divide 4876 by 23.

PROCESS.

23)4876(212, *Quotient.*
46
27
23
46
46

Divide 48 hundreds by 23, and write the result, 2 hundreds, at the right of the dividend for the first or hundreds' term of the quotient. Multiply the divisor (23) by this quotient term, and subtract the product, 46 hundreds, from 48 hundreds. To the remainder, 2 hundreds, annex the 7 tens of the dividend, giving 27 tens for the *second partial dividend.*

Divide 27 tens by 23, and write the result, 1 ten, for the tens' term of the quotient. Multiply 23 by this 1 ten, and subtract the product, 23 tens, from 27 tens. To the remainder, 4 tens, annex the 6 units of the dividend, giving 46 units for the *third partial dividend.*

ILLUSTRATIVE PROCESS.

23	4600	200
23	230	10
23	46	2

23)4876(212

Divide 46 units by 23, and write the result, 2, for the units' term of the quotient. Multiply 23 by 2, and take the result from 46. The quotient is 212.

(2) PROCESS.

212, *Quotient.*
23)4876
46
27
23
46
46

The quotient may be written *over the dividend,* as is shown at the left. Each quotient figure is written over the last figure of the partial dividend that gives it. This is similar to the method in short division. In short division the quotient is written *under* the dividend, and in long division it is written *over* the dividend.

NOTE.—In this and the next 14 problems, each term of the quotient may be determined *by dividing the left-hand term of the partial dividend by the left-hand term of the divisor.*

3. Divide 4664 by 22.
4. Divide 2825 by 25.
5. Divide 4686 by 22.
6. Divide 68952 by 221.
7. Divide 3813 by 123.
8. Divide 63336 by 203.
9. Divide 446886 by 2013.
10. Divide 678273 by 2113.
11. 549661 by 1043.
12. 494505 by 2035.
13. 457632 by 2043.
14. 4654288 by 3056.
15. 886784 by 2048.
16. 8617504 by 2044.
17. 148895 by 307.
18. 16080 by 67.

Since 67 is not contained in the number denoted by the first two left-hand terms of the dividend, take 160 *hundreds* for the *first partial dividend.*

The last partial dividend is 0, and the last quotient figure is 0.

(18) PROCESS.

```
67)16080(240, Quotient.
   134
   ---
    268
    268
    ---
      0
```

19. Divide 312048 by 24.
20. Divide 79272 by 36.
21. Divide 1625130 by 65.
22. Divide 25272 by 36.
23. Divide 302526 by 63.
24. Divide 64347 by 267.
25. Divide 49179 by 507.
26. Divide 54054 by 91.
27. 1200484 by 79.
28. 447125 by 73.
29. $21170 by $365.
30. $28431 by $351.
31. $212992 by $52.
32. 171814 by 634.
33. 120280 by 485.
34. 34137 by 84.

Since the divisor is not contained in the second partial dividend (53 tens), write 0 in the tens' place in the quotient, and annex the 7 units for a *third* partial dividend.

The remainder may also be written over the divisor at the right of the quotient; thus, $406\frac{33}{84}$.

(34) PROCESS.

```
84)34137(406, Quotient.
   336
   ---
     537
     504
     ---
      33, Remainder.
```

35. Divide 24399 by 48.
36. Divide 467034 by 806.
37. Divide 2845007 by 5728.
38. Divide 215607 by 1806.
39. Divide 1423685 by 6785.
40. 1604083 by 2088.
41. 85176 by 168.
42. 268272 by 608.
43. 569536 by 704.
44. 4859663 by 809.

45. Divide 187500 by 250.
46. Divide 69120 by 480.
47. Divide 518400 by 9600.
48. Divide 562500 by 7500.
49. Divide 588600 by 450.
50. Divide 398560 by 940.
51. Divide 126400 by 15800.
52. 1180800 by 480.
53. 1383242 by 406.
54. 7394760 by 36072.
55. 2726394 by 409.
56. $875380 by $253.
57. $4274724 by $606.
58. $190625 by $625.

59. Divide 350 by 10.

FIRST PROCESS.

```
10)350(35, Quotient.
   30
   ---
    50
    50
   ---
```

SECOND PROCESS.

```
1|0)35|0
    ----
      35, Quotient.
```

By comparing these two processes, it is seen that 350 is divided by 10 by cutting off the right-hand figure. The cutting-off of the right-hand figure removes each of the other figures one place to the right, and thus divides their value by 10.

The cutting-off of the two right-hand figures divides a number by 100; the three right-hand figures, by 1000, etc.

Pupils should use the *second* process.

60. Divide 2800 by 10; by 100.
61. Divide 45600 by 10; by 100.
62. Divide 187000 by 1000; by 100.
63. Divide 384050 by 100; by 1000.
64. Divide 230045 by 1000; by 10000.
65. Divide 450860 by 10000; by 1000.

Where is the divisor written in short division? Where is the quotient written?

How is each dividend after the first formed? How may the final remainder be written? What is the proof?

Where is the divisor written in long division? Where is the quotient written?

How is each partial dividend after the first formed? How may the final remainder be written? What is the proof?

How may any number be divided by 10, 100, 1000, etc.?

PROBLEMS IN DIVISION.

ORAL PROBLEMS.

1. If a man walk 3 miles an hour, how long will it take him to walk 15 miles?

Solution.—It will take as many hours as 3 miles are contained times in 15 miles, which is 5: it will take 5 hours.

2. In an orchard there are 16 trees, in rows of 4 trees each: how many rows in the orchard?

3. How many ranks of 4 soldiers each will 24 soldiers make? 32 soldiers? 40 soldiers?

4. A man planted 30 peach-trees in rows, setting 5 trees in each row: how many rows did they make?

5. Mary is reading 5 chapters a day: how long will it take her to read 45 chapters? 60 chapters?

6. A boy had 50 peach-stones, which he planted in rows of 5 each: how many rows did he plant?

7. If 6 chairs make a set, how many sets will 36 chairs make? 48 chairs? 60 chairs? 72 chairs?

8. There are 7 days in a week: how many weeks in 49 days? In 56 days? 63 days? 77 days?

9. There are 6 feet in a fathom: how many fathoms in 54 feet? In 60 feet? 72 feet? 66 feet?

10. There are 8 quarts in a peck: how many pecks in 72 quarts? 48 quarts? 64 quarts?

11. If a steamer run 8 miles an hour, in how many hours will it run 80 miles? 72 miles?

12. If 8 desks make a row, how many rows will 48 desks make? 56 desks?

13. If a man work 8 hours a day, in how many days will he work 72 hours? 96 hours?

14. How long will it take a steamer to make a trip of 81 miles if it run 9 miles an hour?

15. If 9 words fill a line, how many lines will 72 words fill? 81 words? 90 words?

16. How many chairs, at $4 apiece, can be bought for $36? For $40? $48? $28? $32?

17. How many pairs of boots, at $5 a pair, can be bought for $35? For $55? $45? $25? $40?

18. How many plows, at $6 apiece, can be bought for $48? For $54? $60? $72? $36? $42?

19. A woman buys a sewing-machine for $56, and is to pay for it in payments of $4 a month: in how many months will she pay for it?

20. A seamstress earns $12 a week, and spends $5 a week for board and $2 a week for other expenses: in how many weeks will she save $50?

WRITTEN PROBLEMS.

21. At $3 a bushel, how many bushels of wheat can be bought for $963? For $639?

22. In how many hours can a man walk 396 miles, if he walk at the rate of 3 miles an hour?

23. If a man earn $2 a day, how long will it take him to earn $360? $350? $972?

24. The issues of a daily paper for 6 days were 48600 copies: how many copies did it issue daily?

25. If 4 bushels of wheat will make a barrel of flour, how many barrels will 972 bushels make?

26. If a man earn $4 a day, how many days will it take him to earn $1584? $2140? $4356?

27. There are 36 inches in a yard: how many yards are there in 792 inches? In 2376 inches?

28. A bushel of corn weighs 56 pounds: how many bushels in 244160 pounds of corn?

29. A hogshead of molasses contains 63 gallons: how many hogsheads in 47880 gallons?

30. If 72 books can be packed in a box, how many boxes will it take to hold 174960 books?

31. How many farms of 156 acres each can be sold from a tract of land containing 7332 acres?

32. If a vessel sail, on an average, 47 miles a day, how long will it take it to sail 2303 miles?

33. There are 365 days in a common year: how many common years are there in 901550 days?

34. A pipe discharges 94 gallons in an hour: in how many hours will it empty a cistern holding 3384 gallons of water?

35. In 1 week there are 168 hours: how many weeks in 85008 hours? In 254016 hours?

36. A merchant's sales in 1889 amounted to $349848: what were his average sales per month?

37. If a garrison consume 648 pounds of bread in a day, how long will 134136 pounds last it?

38. If the average daily receipts of a ferry-boat be $275, in how many days will its receipts be $165825?

39. A barrel of beef contains 200 pounds: how many barrels will contain 128000 pounds?

40. There are 480 sheets of paper in a ream: how many reams in 1296000 sheets?

Division is the process of finding how many times one number is contained in another.

Division may also be considered as the process of finding one of the equal parts of a number.

A number is contained in another number as many times as it is taken to produce it; hence *division is the inverse of multiplication.*

One number is contained in another number as many times as it can be taken from it; and hence *division is a short method of finding how many times one number may be taken from another.*

In *long division* the partial products and partial dividends are written; in *short division* they are not written, but are found mentally.

FACTORS AND DIVISORS.

FACTORS.

NOTE.—The terms *number*, *factor*, and *divisor*, used in these exercises, denote *integers*.

ORAL EXERCISES.

1. WHAT two numbers multiplied together will produce 6? 10? 14? 15? 21? 25? 27?

2. What two numbers multiplied together will produce 12? 16? 18? 20? 24? 30? 36? 40?

The numbers which, multiplied together, will produce a number are called its *factors*.

3. What are the factors of 15? 25? 35? 45? 55? 21? 27? 36? 42? 63? 56? 60?

4. What are the factors of 20? 28? 32? 48? 54? 56? 50? 63? 64? 72? 81? 70? 90? 100?

5. Give all the pairs of factors of 12.

ANSWER.—12 is 3 times 4 or 6 times 2.

6. Give all the pairs of factors of 18; 20; 24; 30.

7. What three numbers multiplied continuously will produce 12?

ANSWER.—2, 2, and 3; $2 \times 2 \times 3 = 12$.

8. What three numbers multiplied continuously will produce 18? 20? 27? 28? 30? 40? 36?

9. What three numbers multiplied continuously will produce 42? 45? 50? 70? 60? 48? 54? 63?

A *composite* number is the product of two other numbers. Thus, 6 is composite, being the product of 2 and 3.

NOTE.—A composite number may also be the product of more than two numbers. Thus, 30 is the product of 2, 3, and 5.

10. Name all the composite numbers from 0 to 10; from 10 to 20; 20 to 30; 30 to 40.

A *prime* number is not the product of two other numbers. Thus, 3, 7, and 11 are prime numbers.

Every number is the product of itself and 1.

11. Name all the prime numbers from 0 to 10; from 10 to 20; 20 to 30; 30 to 40.

12. Give all the prime factors of 12, not including 1.

ANSWER.—2, 2, and 3; $2 \times 2 \times 3 = 12$.

13. Give all the prime factors of 16; 18; 20; 24; 28; 30; 32; 36; 40; 45; 42; 49; 50.

14. Of what number are 2, 2, and 3 the prime factors? 2, 3, and 3? 2, 3, and 5? 3, 5, and 7?

WRITTEN EXERCISES.

15. What are the prime factors of 60?

16. Of 32? 27? 45? 50? 56?

17. Of 42? 54? 66? 49? 54?

18. Of 56? 63? 64? 72? 70?

19. Of 84? 88? 90? 96? 100?

20. Of 80? 81? 65? 77? 108?

21. Of 28? 48? 70? 75? 200?

22. Of 40? 60? 150? 125? 250?

(15) PROCESS.

$$\begin{array}{r} 2\underline{)60} \\ 2\underline{)30} \\ 3\underline{)15} \\ 5 \end{array}$$

2, 2, 3, 5, *Prime factors.*

23. What prime factors are common to 54 and 72?

PROCESS.

The common factors may be designated by drawing a line through them.

$$\begin{array}{l} 54 = \cancel{2} \times \cancel{3} \times \cancel{3} \times 3 \\ \underline{72 = \cancel{2} \times 2 \times 2 \times \cancel{3} \times \cancel{3}} \end{array}$$

2, 3, 3, *Common factors.*

24. What prime factors are common to 42 and 54? 60 and 90? 84 and 108? 54 and 81? 70 and 105?

25. What prime factors are common to 63 and 54? 72 and 48? 30 and 150? 100 and 125?

DIVISORS.

1. What numbers are divisors of 15? 18? 21? 28? 30? 32? 36? 40? 42? 63? 54? 72?

2. What is the greatest divisor of 18? 30? 45? 40? 42? 50? 55? 60? 63? 70? 75?

3. What number is a divisor of both 10 and 24? 21 and 35? 32 and 56? 35 and 63? 50 and 75?

A divisor of two or more numbers is called a *common divisor*. Thus, 9 is a common divisor of 45 and 63.

4. What is a common divisor of 25 and 30? 63 and 72? 81 and 90? 72 and 96? 56 and 80?

5. What is a common divisor of 50 and 60? 33 and 55? 42 and 54? 45 and 75? 100 and 150?

6. What number will 2, 3, and 5 each divide?

ANSWER.—2, 3, and 5 will each divide 30.

7. What number will 2, 4, and 5 each divide?

8. What number will 3, 5, and 6 each divide?

9. What is the smallest number that 3 and 5 will each divide? 6 and 8? 5 and 6? 6 and 9? 8 and 10?

10. What is the smallest number that 4 and 5 will each divide? 5 and 7? 3 and 10? 6 and 7? 7 and 8?

A Factor of a number is one of the two or more numbers which, multiplied together, will produce it.

The multiplicand and multiplier are factors of their product.

The Divisor of a number is any number that will exactly divide it.

Every divisor of a number is a factor, and every factor is a divisor. The terms factor and divisor differ only in their origin, the former implying *multiplication* and the latter *division*. A divisor of a number is also called its *measure*.

REVIEW PROBLEMS.

1. The sum of two numbers is 15, and one of the numbers is 6: what is the other number?

Suggestion.—The other number is 15 less 6, which is 9.

2. The sum of two numbers is 19, and one of the numbers is 12: what is the other number?

3. The difference of two numbers is 8, and the smaller number is 9: what is the larger number?

4. The difference of two numbers is 7, and the smaller number is 8: what is the larger number?

5. The product of two numbers is 56, and one of the numbers is 7: what is the other number?

6. The product of two numbers is 72, and one of the numbers is 8: what is the other number?

7. The quotient is 6, and the divisor is 8: what is the dividend?

8. The quotient is 9, and the divisor is 12: what is the dividend?

9. A farmer had 23 sheep; he sold 9 of them and 6 were killed by dogs: how many sheep had he left?

10. At $7 a barrel, how many barrels of flour can be bought for $84? For $140? $210?

11. If a vessel sail 72 miles in 9 hours, how many miles does it sail per hour?

Solution.—If a vessel sail 72 miles in 9 hours, in 1 hour it sails $\frac{1}{9}$ of 72 miles, which is 8 miles. It sails 8 miles an hour.

12. If a man can dig 90 rods of ditch in 10 days, how many rods can he dig in 1 day?

13. If 9 barrels of flour cost $63, how much does 1 barrel cost? 5 barrels? 6 barrels?

14. If a stage-coach runs 108 miles in 12 hours, how many miles does it run in 1 hour? In 5 hours?

15. Nine dictionaries cost $72: how much is that apiece?

16. How many barrels of flour at $8 a barrel will pay for 24 yards of carpeting at $2 a yard?

17. How many tons of coal at $9 a ton will pay for 15 cords of wood at $6 a cord?

18. A grocer bought 7 barrels of flour at $6 a barrel: for how much must he sell the lot to gain $14? For how much per barrel?

19. When oranges are sold at 5 cents apiece and lemons at 3 cents apiece, how many cents will buy 6 oranges and 8 lemons?

20. John has 6 marbles, Willis has 4 times as many as John, and Charles has as many as both John and Willis: how many marbles has Charles?

21. Mary bought 6 yards of muslin at 8 cents a yard, 4 yards of braid at 3 cents a yard, and 3 spools of thread at 5 cents apiece, and gave the shop-keeper a silver dollar: how much change did she receive?

22. A boy bought 15 quarts of chestnuts for $1 and sold them at 10 cents a quart: how much did he make?

23. A grocer bought 5 gallons of maple syrup at 60 cents a gallon and sold it at 80 cents a gallon: how much did he make?

24. How many times will a hoop 5 feet in circumference turn round in rolling 75 feet? 100 feet?

25. How many times will a wheel 8 feet in circumference turn round in running 200 feet?

WRITTEN PROBLEMS.

26. The greater of two numbers is 4056, and their difference is 3650: what is the less number?

27. The subtrahend is 34203, and the remainder is 8706: what is the minuend?

28. The divisor is 534, and the quotient 43: what is the dividend?

29. The product is 15288 and the multiplier 98: what is the multiplicand?

30. The dividend is 5292, and the divisor is 63: what is the quotient?

31. The product of two numbers is 5328, and one of the numbers is 148: what is the other number?

32. Multiply the sum of 486 and 392 by 94.

33. Divide the product of 48 and 24 by 36, and multiply the quotient by 50.

34. A merchant bought 35 yards of cloth for $56, and sold it at $2 a yard: how much did he gain?

35. A drover bought 145 horses at $94 each and sold the lot for $15500: how much did he gain?

36. A grocer bought 94 barrels of apples at $2.50 a barrel and sold them at $3 a barrel: how much did he gain?

37. A man's income is $3500 a year; he pays $450 a year for house-rent, $150 for taxes, $350 for hired help, and $540 for other expenses: how much are his expenses? How much has he left each year?

38. A man bought 80 acres of land at $35 an acre, paid $325 for improvements, and then sold it for $3750: how much did the land and improvements cost? How much did he gain?

39. The earnings of a coal-mine averaged $17560 a month and its expenses $9455 a month: what were its net earnings in a year?

40. A widow has three sons and two daughters; the oldest son earns $30 a month, the second son $24 a month, the youngest son $10 a month, and each of the daughters $15 a month: how much do her five children earn in 1 month? How much in 12 months?

41. The distance from New-York to San Francisco by rail is 3424 miles: in how many hours will a train run from one city to the other running at the rate of 32 miles an hour?

42. A grocer sold 16 firkins of butter containing 95 pounds each, at 25 cents a pound: how much did he receive? The butter cost him $316: how much did he gain?

43. A man receives a salary of $1500 a year; he pays $20 a month for a house, $10 a month for help, $35 a month for meat and groceries, $250 a year for clothing, and $120 a year for other expenses: how much are his expenses each year? How much has he left?

44. A man starting on a journey took $200; he paid for railroad fare, $67; for berth in sleeper, 4 days, $2 a day; for hotel bills, 15 days, $3 a day; and for other expenses, $25: how much money had he left?

QUESTIONS FOR REVIEW.

What is addition? What is the result called? How many ones does the sum contain?

What is meant by like numbers? Give examples. What kind of numbers can be added?

What is subtraction? What is the result called? Which number is the minuend? Which the subtrahend? What kind of numbers can be subtracted?

What is multiplication? What is the result called? Which number is the multiplicand? Which is the multiplier?

How may the product be obtained by addition? How may any number be multiplied by 10, 100, 1000, etc.?

What is division? What is the result called? Which number is the dividend? Which is the divisor? What is the remainder?

What is the distinction between long division and short division? When may short division be used?

Where is the quotient written in short division? Where is the quotient written in long division? How is each partial product, after the first, formed?

How may any number be divided by 10, 100, 1000, etc.?

What is meant by the factors of a number? When is a factor called a prime factor?

What is a divisor of a number? What is a common divisor of two or more numbers?

PART III.

FRACTIONS.

THE IDEA OF A FRACTION DEVELOPED.

1. IF a melon be cut into two *equal* pieces, what part of the melon will one piece be?

2. How many halves in a melon? How many halves in any thing?

3. If a melon be cut into four equal pieces, what part of the melon will one piece be? Two pieces? Three pieces? Four pieces?

4. How many fourths in an apple? How many fourths in any thing?

5. Which is the greater, one half or one fourth of an apple? How many fourths equal one half?

6. If an apple be cut into three equal pieces, what part of the apple will one piece be? Two pieces?

7. How many thirds in an apple? How many thirds in any thing?

8. If a cake be cut into six equal pieces, what part of the cake will one piece be? Two pieces? Three pieces? Four pieces? Five pieces?

9. How many sixths in a cake? How many sixths in any thing?

10. Which is the greater, one third or one sixth of a cake? One fourth or one half?

11. One thing of any kind is a *unit.* How many halves in a unit? How many thirds? How many fourths? How many fifths? How many sixths?

12. What is meant by one third?

Ans.—*One third is one of the three equal parts of a unit.*

13. What is meant by two thirds? One fourth? Three fourths? One sixth? Three sixths?

Such parts of a unit as two thirds, three fourths, one sixth, etc., are called *fractions.*

A fraction is expressed by two numbers, one being called the *numerator* and the other the *denominator.*

The denominator is the number of equal parts into which the unit is divided. The numerator is the number of equal parts taken.

A fraction is expressed in figures by writing the numerator over the denominator; thus, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.

Read the following fractions, and in each case tell into how many equal parts the unit is divided, and how many parts are taken:—

14.	**15.**	**16.**	**17.**	**18.**	**19.**
$\frac{3}{4}$	$\frac{6}{7}$	$\frac{5}{12}$	$\frac{5}{17}$	$\frac{6}{25}$	$\frac{20}{9}$
$\frac{5}{6}$	$\frac{9}{18}$	$\frac{8}{15}$	$\frac{13}{18}$	$\frac{13}{30}$	$\frac{5}{16}$
$\frac{7}{8}$	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{10}{19}$	$\frac{15}{20}$	$\frac{11}{12}$

20. Which is the greater, two thirds or a unit? Five thirds or a unit? Three thirds or a unit?

21. Which is the greater, three fourths or a unit? Four fourths or a unit? Six fourths or a unit?

When the value of a fraction is less than a unit it is a *proper fraction.* When its value is a unit or more than a unit, it is an *improper fraction.*

The numerator of a proper fraction is less than the denominator; and the numerator of an improper fraction is equal to or greater than the denominator.

22. Write in figures two fifths; nine fifths; seven tenths; seven sevenths; ten ninths; six thirteenths; eleven tenths; fifteen sixths; three eighths.

23. Which of the above fractions are proper fractions? Which are improper fractions?

A Fraction is one or more of the equal parts of a unit.

The numerator and denominator are called the *terms* of a fraction.

REDUCTION OF FRACTIONS.

INTEGERS AND MIXED NUMBERS REDUCED TO FRACTIONS.

1. How many thirds in an apple? How many in 2 apples?

2. How many fourths in an apple? How many in 3 apples?

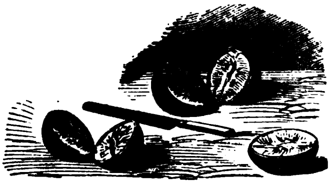
3. How many fourths in 2 pears? In 3 pears? In 4 pears?

Solution.—In 1 pear there are 4 fourths, and in 3 pears there are 3 times 4 fourths, which is 12 fourths. There are 12 fourths in 3 pears.

4. How many sixths in 3 oranges? In 5 oranges?

5. How many fifths in 3? 2? 5? 8? 10?

6. How many eighths in 4? 2? 6? 8? 10?



7. How many halves in 2 and 1 half oranges?

Solution. — In 2 oranges there are twice 2 halves, which is 4 halves, and 4 halves and 1 half are 5 halves. There are 5 halves in 2 and 1 half oranges.

8. How many fourths in 2? In 2 and 3 fourths?

9. How many thirds in 5? In 5 and 2 thirds?

10. How many thirds in 7 and 2 thirds?

11. How many sixths in 5? In $5\frac{2}{6}$? $3\frac{5}{6}$? $6\frac{1}{6}$?

12. How many tenths in $6\frac{3}{10}$? $5\frac{3}{10}$? $7\frac{4}{10}$? $8\frac{7}{10}$?

13. How many fifths in $6\frac{2}{5}$? $8\frac{3}{5}$? $12\frac{4}{5}$? $10\frac{3}{5}$? $9\frac{2}{5}$?

14. How many eighths in $5\frac{3}{8}$? $7\frac{4}{8}$? $9\frac{5}{8}$? $10\frac{7}{8}$? $12\frac{5}{8}$?

An integer and a fraction united as one number is a *mixed number.* $12\frac{1}{2}$ and $16\frac{2}{3}$ are mixed numbers.

WRITTEN EXERCISES.

15. Solve examples 13 and 14 above on slate or paper.

16. Reduce 157 to ninths; $157\frac{7}{9}$ to ninths.

PROCESS.

$$\begin{array}{r} 157 \\ 9 \\ \hline 1413 \end{array} \quad 157 = \frac{1413}{9}, \textit{ Ans.}$$

PROCESS.

$$\begin{array}{r} 157\frac{7}{9} \\ 9 \\ \hline 1420 \end{array} \quad 157\tfrac{7}{9} = \frac{1420}{9}, \textit{ Ans.}$$

17. Reduce $96\frac{5}{8}$ to eighths; $75\frac{3}{4}$ to fourths.

18. Reduce $46\frac{7}{9}$ to ninths; $55\frac{2}{3}$ to thirds.

19. Reduce $63\frac{3}{11}$ to elevenths; $63\frac{3}{10}$ to tenths.

20. Reduce $53\frac{5}{9}$ to an improper fraction.

Suggestion.—Reduce the mixed number, $53\frac{5}{9}$, to ninths.

21. Reduce to an improper fraction $33\frac{4}{5}$; $85\frac{5}{9}$; $36\frac{7}{8}$; $48\frac{7}{10}$; $236\frac{4}{5}$; $75\frac{9}{20}$; $365\frac{3}{4}$.

22. Reduce to an improper fraction $109\frac{3}{6}$; $153\frac{4}{5}$; $204\frac{3}{10}$; $200\frac{7}{12}$; $115\frac{41}{100}$; $108\frac{15}{22}$; $214\frac{9}{20}$; $330\frac{16}{25}$.

FRACTIONS REDUCED TO INTEGERS OR MIXED NUMBERS.

1. How many pears in 4 half-pears? In 6 half-pears? In 7 half-pears? In 8 half-pears? In 9 half-pears?

2. How many pears in 10 half-pears? In 12 half-pears? In 16 half-pears?

3. How many days in 11 half-days?

SOLUTION.—In 11 half-days there are as many days as 2 half-days are contained times in 11 half-days, which is 5½ times. There are 5½ days in 11 half-days.

4. How many pints in 14 half-pints? In 17 half-pints? In 21 half-pints? In 25 half-pints?

5. How many yards in 18 thirds of a yard? In 19 thirds of a yard? In 22 thirds of a yard?

6. How many ones in 25 fourths? In 30 fourths?

7. How many ones in 25 eighths? In 30 eighths?

8. How many ones in $\frac{45}{9}$? $\frac{56}{9}$? $\frac{75}{9}$? $\frac{84}{9}$? $\frac{90}{9}$?

9. How many ones in $\frac{17}{4}$? $\frac{27}{5}$? $\frac{33}{8}$? $\frac{45}{7}$? $\frac{56}{10}$?

WRITTEN EXERCISES.

10. Reduce $\frac{177}{8}$ to a mixed number.

PROCESS: $\frac{177}{8} = 177 \div 8 = 22\frac{1}{8}$, *Ans.*

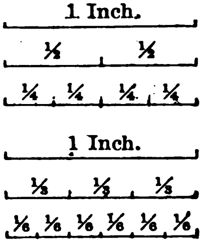
Divide the numerator by the denominator.

Reduce to an integer or mixed number:—

11. $\frac{65}{9}$
12. $\frac{94}{8}$
13. $\frac{105}{9}$
14. $\frac{75}{7}$
15. $\frac{207}{12}$
16. $\frac{125}{25}$
17. $\frac{160}{15}$
18. $\frac{312}{15}$
19. $\frac{221}{20}$
20. $\frac{322}{16}$
21. $\frac{256}{32}$
22. $\frac{312}{30}$
23. $\frac{160}{80}$
24. $\frac{420}{21}$
25. $\frac{405}{50}$
26. $\frac{448}{21}$
27. $\frac{885}{25}$
28. $\frac{214}{35}$
29. $\frac{256}{16}$
30. $\frac{305}{30}$
31. $\frac{192}{12}$
32. $\frac{625}{25}$
33. $\frac{625}{40}$
34. $\frac{245}{35}$
35. $\frac{1000}{75}$

FRACTIONS REDUCED TO LOWER TERMS.

1. How many half-inches in 2 fourths of an inch? In 4 fourths of an inch?



2. How many halves in 2 fourths? In 4 fourths? In 6 fourths?

3. How many thirds of an inch in 2 sixths of an inch? In 4 sixths? In 6 sixths?

4. How many thirds in 2 sixths? In 4 sixths? In 6 sixths?

5. How many fourths in 4 eighths? In 6 eighths? In 8 eighths?

Solution.—In 2 eighths there is 1 fourth, in 6 eighths there are 3 fourths, and in 8 eighths there are 4 fourths.

6. How many fifths in 2 tenths? In 6 tenths?
7. How many fifths in $\frac{6}{15}$? $\frac{9}{15}$? $\frac{12}{15}$?
8. How many sevenths in $\frac{6}{21}$? $\frac{12}{21}$? $\frac{15}{21}$?
9. How many eighths in $\frac{8}{32}$? $\frac{16}{32}$? $\frac{24}{32}$? $\frac{28}{32}$?
10. Reduce $\frac{9}{12}$, $\frac{8}{16}$, and $\frac{10}{20}$ each to fourths.
11. Reduce $\frac{18}{21}$, $\frac{25}{35}$, and $\frac{16}{28}$ each to sevenths.

When a fraction is reduced to smaller terms, as $\frac{9}{18}$ to $\frac{3}{6}$, it is reduced to *lower terms*.

A fraction is reduced to lower terms by dividing both numerator and denominator by the same number.

12. Reduce to lower terms $\frac{20}{50}$; $\frac{16}{20}$; $\frac{18}{30}$.

To Teachers.—Show that the dividing of both terms of a fraction by the same number does not change its value.

13. Reduce to lower terms $\frac{8}{12}$; $\frac{12}{36}$; $\frac{20}{50}$; $\frac{15}{40}$.

When no integer except 1 will divide both numerator and denominator of a fraction, it is in its *lowest terms*.

14. Reduce to lowest terms $\frac{25}{50}$; $\frac{18}{30}$; $\frac{21}{42}$; $\frac{32}{56}$; $\frac{36}{42}$.
15. Reduce to lowest terms $\frac{15}{45}$; $\frac{10}{30}$; $\frac{18}{45}$; $\frac{15}{60}$; $\frac{36}{60}$.

WRITTEN EXERCISES.

16. Reduce $\frac{63}{84}$ to its lowest terms.

PROCESS.

$$\frac{63 \div 3}{84 \div 3} = \frac{21}{28}\,;$$

$$\frac{21 \div 7}{28 \div 7} = \frac{3}{4}, \quad \textit{Ans.}$$

Reduce $\frac{63}{84}$ to lower terms by dividing both terms by 3; next reduce $\frac{21}{28}$ to lower terms by dividing both terms by 7. The resulting fraction, $\frac{3}{4}$, is in its lowest terms.

17. Reduce the fractions in examples 12, 13, 14, and 15 above to lowest terms.

18. Reduce to lowest terms $\frac{64}{72}$; $\frac{36}{108}$; $\frac{72}{144}$.

19. $\frac{56}{84}$
20. $\frac{72}{108}$
21. $\frac{75}{100}$
22. $\frac{96}{108}$
23. $\frac{105}{150}$
24. $\frac{125}{175}$
25. $\frac{72}{96}$
26. $\frac{32}{160}$
27. $\frac{108}{144}$
28. $\frac{144}{432}$
29. $\frac{225}{300}$
30. $\frac{39}{143}$
31. $\frac{125}{625}$
32. $\frac{64}{256}$
33. $\frac{72}{576}$

To TEACHERS.—The method of reducing fractions to their lowest terms by first finding the *greatest common divisor* need not be taught to young pupils. All of the fractions in this book may be readily reduced by using *obvious* divisors, and pupils may be trained to see the greatest divisor. Thus, $\frac{63}{84}$ may be reduced to lowest terms by dividing both terms by 21.

FRACTIONS REDUCED TO HIGHER TERMS.

1. How many fourths of an orange in 1 half? In 2 halves?

2. How many eighths of an orange in 1 fourth of an orange? In 2 fourths? In 3 fourths?

3. How many eighths in 1 fourth? In 3 fourths?

SOLUTION.—One fourth is 2 eighths, and 3 fourths are 3 times 2 eighths, which is 6 eighths.

4. How many ninths in 1 third? In 2 thirds?
5. How many tenths in $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{5}$? $\frac{6}{5}$?
6. How many twelfths in $\frac{2}{6}$? $\frac{4}{6}$? $\frac{5}{6}$? $\frac{3}{6}$? $\frac{7}{6}$?
7. Change $\frac{3}{4}$ and $\frac{5}{6}$ each to twelfths.
8. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{9}$ each to eighteenths.
9. Change $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ each to twenty-fourths.
10. Change $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{7}{15}$ each to thirtieths.
11. Change $\frac{3}{7}$, $\frac{11}{14}$, and $\frac{1}{2}$ each to twenty-eighths.
12. Reduce $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$ each to twenty-fourths.

WRITTEN EXERCISES.

13. Reduce $\frac{17}{30}$ to sixtieths.

One thirtieth is 2 sixtieths, and 17 thirtieths are 17 times 2 sixtieths, which is 34 sixtieths. This is the same as dividing 60 by 30 and multiplying both terms of $\frac{17}{30}$ by 2, the quotient.

PROCESS.

$$60 \div 30 = 2$$

$$\frac{17 \times 2}{30 \times 2} = \frac{34}{60}, \textit{ Ans.}$$

To Teachers.—Show that the multiplying of both terms of a fraction by the same number does not change its value.

14. Solve examples 9, 10, 11, and 12 above on paper.
15. Reduce $\frac{19}{24}$ to ninety-sixths; to seventy-seconds.
16. Reduce $\frac{3}{7}$ and $\frac{11}{21}$ each to forty-seconds.
17. Reduce $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{5}{6}$ each to twenty-fourths.

When two or more fractions have the same denominator they are called *like fractions.*

The denominator of like fractions is called their *common denominator.*

18. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to like fractions.

PROCESS: $\frac{5}{6} = \frac{20}{24}$; $\frac{7}{8} = \frac{21}{24}$; $\frac{11}{12} = \frac{22}{24}$.

Change the fractions to *twenty-fourths.*

19. Reduce $\frac{7}{12}$, $\frac{5}{18}$, and $\frac{7}{9}$ to like fractions.

Suggestion.—Change the fractions to *thirty-sixths.*

20. Reduce to like fractions $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{7}{15}$.

21. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$
22. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{9}$
23. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{8}{15}$
24. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$
25. $\frac{5}{1}$, $\frac{3}{7}$, $\frac{7}{12}$
26. $\frac{1}{6}$, $\frac{5}{12}$, $\frac{7}{24}$
27. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{4}{15}$, $\frac{11}{30}$
28. $\frac{4}{9}$, $\frac{5}{6}$, $\frac{11}{18}$, $\frac{13}{36}$
29. $\frac{4}{5}$, $\frac{9}{10}$, $\frac{14}{25}$, $\frac{33}{50}$

NOTE.—The common denominator should be found by inspection. It is not necessary to find the least common multiple.

QUESTIONS FOR REVIEW.

What is a fraction? How many numbers are used to express a fraction? What are they called?

What does the denominator of a fraction denote? What does the numerator denote?

How is a fraction expressed by figures? When is a fraction proper? When is it improper? Give examples.

What is the value of a proper fraction compared with its unit? The value of an improper fraction?

What is a mixed number? Change $\frac{13}{4}$ to a mixed number.

When is a fraction in its lowest terms? Give examples.

When are two or more fractions called like fractions? What is the denominator of like fractions called?

ADDITION OF FRACTIONS.

1. A boy gave $\frac{1}{4}$ of a melon to his brother, $\frac{1}{4}$ to his sister, and $\frac{1}{4}$ to a playmate: what part did he give away?

How many fourths are $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$?

2. A grocer sold $\frac{1}{8}$ of a cheese to one customer, $\frac{2}{8}$ to another, and $\frac{3}{8}$ to another: what part of it did he sell?

How many eighths are $\frac{1}{8}+\frac{2}{8}+\frac{3}{8}$?

3. How many sixths are $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{5}{6}$? $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{1}{6}$?

4. A boy gave $\frac{1}{2}$ of his money for a knife, and $\frac{1}{3}$ of it for a ball: what part of his money did he spend?

SOLUTION.—Change $\frac{1}{2}$ and $\frac{1}{3}$ to sixths. $\frac{1}{2}$ is $\frac{3}{6}$ and $\frac{1}{3}$ is $\frac{2}{6}$; $\frac{3}{6}$ and $\frac{2}{6}$ are $\frac{5}{6}$. The boy spent $\frac{5}{6}$ of his money.

5. How many tenths in $\frac{1}{2}$ and $\frac{3}{5}$? $\frac{4}{5}$ and $\frac{3}{10}$?
6. How many twelfths in $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{3}{4}$ and $\frac{5}{6}$?
7. How many eighths in $\frac{1}{4}$ and $\frac{1}{8}$? $\frac{3}{4}$ and $\frac{3}{8}$?
8. What is the sum of $\frac{1}{3}$ and $\frac{2}{5}$? $\frac{2}{3}$ and $\frac{3}{5}$?

SUGGESTION.—Change the fractions to fifteenths.

9. What is the sum of $\frac{1}{4}$ and $\frac{3}{8}$? $\frac{1}{2}$ and $\frac{2}{3}$?
10. What is the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

WRITTEN EXERCISES.

11. What is the sum of $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, and $\frac{9}{13}$?

PROCESS: $\frac{5}{13}+\frac{6}{13}+\frac{7}{13}+\frac{9}{13}=\frac{27}{13}=2\frac{1}{13}$, *Ans.*

12. What is the sum of $\frac{13}{24}$, $\frac{23}{24}$, $\frac{5}{24}$, and $\frac{19}{24}$?
13. What is the sum of $\frac{17}{30}$, $\frac{13}{30}$, $\frac{23}{30}$, and $\frac{29}{30}$?
14. What is the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$?

PROCESS.

$\frac{5}{6}+\frac{7}{8}+\frac{5}{12}=$
$\frac{20}{24}+\frac{21}{24}+\frac{10}{24}=\frac{51}{24}$
$\frac{51}{24}=2\frac{3}{24}=2\frac{1}{8}$, *Ans.*

Change the fractions to twenty-fourths; add the numerators of the like fractions and write the sum over the common denominator; reduce the resulting improper fraction to a mixed number.

15. What is the sum of $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{15}$?

16. Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.
17. Add $\frac{1}{6}$, $\frac{7}{12}$, $\frac{11}{24}$.
18. Add $\frac{2}{5}$, $\frac{3}{10}$, $\frac{13}{15}$.
19. Add $\frac{1}{4}$, $\frac{3}{5}$, $\frac{17}{20}$.
20. Add $\frac{1}{5}$, $\frac{2}{3}$, $\frac{8}{15}$.
21. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$.
22. Add $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$.
23. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$.
24. Add $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{30}$.
25. Add $\frac{2}{3}$, $\frac{5}{14}$, $\frac{9}{21}$, $\frac{11}{42}$.
26. Add $\frac{2}{5}$, $\frac{3}{4}$, $\frac{3}{10}$, $\frac{13}{20}$.
27. Add $\frac{5}{6}$, $\frac{5}{8}$, $\frac{5}{12}$, $\frac{17}{24}$.
28. Add $\frac{3}{7}$, $\frac{9}{14}$, $\frac{20}{21}$, $\frac{31}{42}$.
29. Add $\frac{9}{10}$, $\frac{11}{15}$, $\frac{3}{20}$, $\frac{23}{30}$.

30. What is the sum of $16\frac{2}{3}$, $18\frac{3}{4}$, and $37\frac{1}{2}$?

PROCESS.

$$\begin{array}{r}
16\frac{2}{3}=16\frac{8}{12}\\
18\frac{3}{4}=18\frac{9}{12}\\
37\frac{1}{2}=37\frac{6}{12}\\
\hline
72\frac{11}{12}\text{, } \textit{Ans.}
\end{array}$$

First add the fractions and then the integers, and combine the results.

$\frac{8}{12}+\frac{9}{12}+\frac{6}{12}=\frac{23}{12}=1\frac{11}{12}$; write the $\frac{11}{12}$ under the fractions and add the 1 with the integers.

31. What is the sum of $65\frac{1}{4}$, 340, $225\frac{1}{2}$, and $408\frac{5}{6}$?

32. Add $45\frac{1}{2}$, $67\frac{3}{4}$, $62\frac{5}{8}$.
33. Add $30\frac{1}{4}$, $66\frac{2}{3}$, $84\frac{3}{4}$.
34. Add $75\frac{1}{2}$, 108, $160\frac{3}{5}$.
35. Add $66\frac{2}{3}$, $305\frac{3}{4}$, $450\frac{5}{6}$.
36. Add $166\frac{2}{3}$, $409\frac{3}{4}$, $3047\frac{1}{2}$.
37. Add $102\frac{6}{7}$, $156\frac{7}{21}$, $905\frac{1}{3}$.
38. Add $30\frac{1}{4}$, $267\frac{1}{2}$, $3230\frac{3}{8}$.
39. Add 440, $90\frac{7}{12}$, $649\frac{7}{8}$.

To add fractions:—

RULE.—*Reduce the fractions to like fractions, add the numerators of the like fractions, and write the sum over the common denominator.*

SUBTRACTION OF FRACTIONS.

1. Albert had $\frac{2}{3}$ of an orange, and he gave $\frac{1}{3}$ to his sister: how many thirds had he left?

How much is $\frac{3}{3}$ less $\frac{1}{3}$? $\frac{3}{3}$ less $\frac{2}{3}$?

2. Charles bought $\frac{3}{4}$ of a pound of nuts, and gave $\frac{1}{4}$ of a pound to Ward: what part of a pound had he left?

How much is $\frac{3}{4}$ less $\frac{1}{4}$? $\frac{3}{4}$ less $\frac{2}{4}$?

3. How much is $\frac{2}{3}$ less $\frac{1}{2}$?

SOLUTION.—Change $\frac{2}{3}$ and $\frac{1}{2}$ to sixths; $\frac{2}{3}$ is $\frac{4}{6}$, $\frac{1}{2}$ is $\frac{3}{6}$; $\frac{4}{6}$ less $\frac{3}{6}$ is $\frac{1}{6}$.

4. How much is $\frac{3}{4}$ less $\frac{1}{2}$? $\frac{3}{4}$ less $\frac{1}{8}$? $\frac{2}{3}$ less $\frac{1}{2}$?
5. How much is $\frac{7}{8}$ less $\frac{3}{4}$? $\frac{7}{8}$ less $\frac{2}{3}$? $\frac{7}{8}$ less $\frac{1}{4}$?
6. How much is $\frac{7}{12}$ less $\frac{1}{3}$? $\frac{7}{12}$ less $\frac{1}{2}$? $\frac{7}{12}$ less $\frac{1}{4}$?
7. How much is $\frac{9}{10}$ less $\frac{3}{4}$? $\frac{9}{10}$ less $\frac{4}{5}$? $\frac{9}{10}$ less $\frac{1}{2}$?
8. How much is $\frac{15}{16}$ less $\frac{5}{8}$? $\frac{15}{16}$ less $\frac{3}{4}$? $\frac{15}{16}$ less $\frac{7}{8}$?
9. How much is $\frac{7}{9}$ less $\frac{1}{6}$? $\frac{8}{15}$ less $\frac{3}{10}$? $\frac{3}{4}$ less $\frac{3}{8}$?

WRITTEN EXERCISES.

10. From $\frac{11}{24}$ take $\frac{7}{24}$.

PROCESS: $\frac{11}{24} - \frac{7}{24} = \frac{4}{24} = \frac{1}{6}$, *Ans.*

11. From $\frac{29}{54}$ take $\frac{11}{54}$; from $\frac{43}{120}$ take $\frac{24}{120}$.

12. From $\frac{11}{12}$ take $\frac{5}{8}$.

PROCESS: $\frac{11}{12} - \frac{5}{8} = \frac{22}{24} - \frac{15}{24} = \frac{7}{24}$, *Ans.*

13. Solve examples 7, 8, and 9 above on slate or paper.

14. Subtract $\frac{3}{8}$ from $\frac{7}{12}$.
15. Subtract $\frac{7}{15}$ from $\frac{9}{10}$.
16. Subtract $\frac{12}{25}$ from $\frac{33}{50}$.
17. Subtract $\frac{5}{18}$ from $\frac{7}{12}$.
18. Subtract $\frac{5}{14}$ from $\frac{8}{21}$.
19. Subtract $\frac{5}{12}$ from $\frac{7}{15}$.

20. From $\frac{3}{5}$ take $\frac{5}{9}$.
21. From $\frac{13}{15}$ take $\frac{7}{10}$.
22. From $\frac{23}{25}$ take $\frac{13}{50}$.
23. From $\frac{7}{8}$ take $\frac{7}{12}$.
24. From $\frac{13}{14}$ take $\frac{11}{21}$.
25. From $\frac{11}{12}$ take $\frac{11}{15}$.

26. From $33\frac{1}{3}$ take $18\frac{3}{4}$.

PROCESS.

$$\begin{array}{r} 33\frac{1}{3} = 33\frac{4}{12} \\ 18\frac{3}{4} = 18\frac{9}{12} \\ \hline 14\frac{7}{12}, \textit{ Ans.} \end{array}$$

First subtract the fractions and then the integers. Since $\frac{9}{12}$ is greater than $\frac{4}{12}$, add $\frac{12}{12}$ to $\frac{4}{12}$, making $\frac{16}{12}$, and then take $\frac{9}{12}$ from $\frac{16}{12}$, writing $\frac{7}{12}$ under the fractions, and adding 1 to the 8 units before subtracting the integers.

27. Subtract $30\frac{1}{4}$ from $66\frac{2}{3}$.
28. Subtract $112\frac{3}{4}$ from $145\frac{1}{2}$.
29. Subtract $250\frac{3}{5}$ from 300.
30. Subtract $130\frac{5}{8}$ from $241\frac{2}{3}$.

31. $45\frac{3}{5}$ from $66\frac{2}{3}$.
32. $90\frac{1}{8}$ from $108\frac{1}{2}$.
33. $105\frac{3}{4}$ from $261\frac{1}{3}$.
34. $166\frac{2}{3}$ from $233\frac{1}{3}$.

To subtract fractions:—

RULE.—*Reduce the fractions to like fractions, subtract the numerator of the subtrahend from the numerator of the minuend, and write the difference over the common denominator.*

PROBLEMS REVIEWING ADDITION AND SUBTRACTION.

N. B.—These problems are to be solved first orally and then on slate or paper.

1. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$ take $\frac{3}{4}$.

2. From $\frac{7}{8}$ take the sum of $\frac{1}{2}$ and $\frac{1}{4}$.

3. The sum of two fractions is $\frac{5}{6}$, and one of the fractions is $\frac{1}{4}$: what is the other fraction?

4. The sum of two mixed numbers is 16⅔, and one of the mixed numbers is 9½: what is the other?

5. A boy spent ⅖ of his money for an arithmetic and ¼ of it for a slate: what part of his money did he spend? What part had he left?

6. A man sold ⅖ of his farm to one neighbor and ⅓ of it to another: what part of the farm did he sell? What part had he left?

7. A boy caught three fishes; the first weighed ¾ of a pound, the second ⅞ of a pound, and the third ½ of a pound: how much did the three fishes weigh?

8. A man bought 1½ acres of ground, and then sold ⅓ of an acre to one man and ½ an acre to another: what part of an acre had he left?

9. A boy earned ½ of a dollar one day and ⅝ of a dollar the next day, and then gave his mother ¾ of a dollar: what part of a dollar did he earn? What part of a dollar had he left?

10. James earns $¾ per day and Henry earns $⅗: how much do they both earn per day? How much does James earn more than Henry?

11. One fourth of a farm is pasture, ⅓ of it is meadow, ⅙ of it is woodland, and the rest of it is in grain: what part of the farm is in grain?

12. A farmer has three stacks of hay; the first contains 3½ tons, the second 4½ tons, and the third 4¾ tons: how much hay in the three stacks? How much hay in the third stack more than in the first?

13. A farmer bought ⅞ of a bushel of clover-seed, and then sold ¼ of a bushel to one neighbor and ⅓ of a bushel to another neighbor: what part of a bushel did he sell? What part had he left?

14. In a certain school ⅖ of the pupils are in the first grade, ⅓ of them in the second grade, and the other pupils are in the third grade: what part of the pupils are in the third grade?

MULTIPLICATION OF FRACTIONS.

INTEGERS MULTIPLIED BY FRACTIONS.

1. If 6 pears be divided equally between 2 boys, what part of the whole will each receive?

What is $\frac{1}{2}$ of 6 pears? $\frac{1}{2}$ of 10 pears?

2. A mother divided 5 apples equally between 2 children: what part of the whole did each receive?

What is $\frac{1}{2}$ of 5 apples?

SUGGESTION.—Take 1 half of 4 apples and 1 half of 1 apple.

3. What is $\frac{1}{3}$ of 9 apples? $\frac{1}{3}$ of 10 apples?
4. What is $\frac{1}{4}$ of 20? $\frac{1}{4}$ of 23? $\frac{1}{4}$ of 30?
5. What is $\frac{1}{5}$ of 15? $\frac{1}{5}$ of 17? $\frac{1}{5}$ of 33?
6. What is $\frac{1}{6}$ of 24? $\frac{3}{6}$ of 24? $\frac{5}{6}$ of 24?
7. What is $\frac{1}{10}$ of 40? $\frac{3}{10}$ of 40? $\frac{7}{10}$ of 40?
8. What is $\frac{2}{3}$ of 18? $\frac{3}{4}$ of 20? $\frac{4}{5}$ of 25? $\frac{5}{6}$ of 30?
9. Multiply 36 by $\frac{3}{4}$; 36 by $\frac{4}{9}$; 36 by $\frac{3}{12}$.

SOLUTION.—$36 \times \frac{3}{4} = \frac{3}{4}$ of 36; $\frac{1}{4}$ of $36 = 9$, and $\frac{3}{4}$ of $36 = 27$.

10. Multiply 56 by $\frac{3}{7}$; by $\frac{5}{8}$; by $\frac{6}{7}$; by $\frac{7}{8}$; by $\frac{3}{4}$.

WRITTEN EXERCISES.

11. Multiply 256 by $\frac{7}{8}$.

PROCESS.

$$(1)\ \begin{array}{r} 8)\underline{256} \\ 32 \\ \underline{\quad 7} \\ 224,\ \textit{Ans.} \end{array} \qquad (2)\ \begin{array}{r} 256 \\ \underline{\quad 7} \\ 8)\underline{1792} \\ 224,\ \textit{Ans.} \end{array}$$

(1) $\frac{7}{8}$ times 256 is 7 times $\frac{1}{8}$ of 256.

(2) Or, $\frac{7}{8}$ times 256 is $\frac{1}{8}$ of 7 times 256.

The pupil should use only one of these methods.

12. Solve examples 9 and 10 above on paper.

13. Multiply 48 by $\frac{5}{8}$.
14. Multiply 63 by $\frac{7}{9}$.
15. Multiply 57 by $\frac{6}{7}$.
16. Multiply 87 by $\frac{4}{5}$.
17. 163 by $\frac{8}{11}$.
18. 300 by $\frac{13}{15}$.
19. 248 by $\frac{5}{6}$.
20. 400 by $\frac{5}{9}$.
21. 520 by $\frac{2}{3}$.
22. 432 by $\frac{7}{12}$.
23. 256 by $\frac{11}{16}$.
24. 625 by $\frac{7}{25}$.

25. Multiply 324 by $6\frac{2}{3}$.

First multiply by the integer and then by the fraction, and add the products.

To TEACHERS.—If preferred, the pupils may first multiply by the fraction and then the integer. It is best to teach beginners but one method.

PROCESS.

$$\begin{array}{r} 324 \\ \underline{6\frac{2}{3}} \\ 1944 \\ \underline{216} \\ 2160, \textit{ Ans.} \end{array}$$

26. Multiply 48 by $16\frac{1}{2}$; 48 by $18\frac{3}{4}$.

27. 72 by $18\frac{3}{4}$.
28. 96 by $23\frac{1}{8}$.
29. 246 by $12\frac{2}{3}$.
30. 406 by $33\frac{1}{3}$.
31. 408 by $56\frac{1}{2}$.
32. 520 by $32\frac{2}{5}$.

FRACTIONS MULTIPLIED BY INTEGERS.

1. What part of a cake is twice 2 eighths of it? 3 times 2 eighths of it?

2. A father gave 3 fourths of an orange to each of 4 children: how many fourths did they all receive?

3. How much is 4 times $\frac{3}{4}$? 6 times $\frac{3}{4}$?

4. If a boy earn 2 thirds of a dollar in a day, how much will he earn in 3 days? In 5 days?

5. How much is 3 times $\frac{2}{3}$? 5 times $\frac{2}{3}$? 6 times $\frac{2}{3}$?

6. How much is 6 times $\frac{3}{5}$? 8 times $\frac{2}{5}$? 9 times $\frac{3}{5}$?

7. How much is 7 times $\frac{5}{8}$? 6 times $\frac{7}{8}$? 8 times $\frac{5}{9}$?

WRITTEN EXERCISES.

8. Solve examples 5, 6, and 7 above on paper.

9. Multiply $\frac{5}{8}$ by 10; $\frac{5}{12}$ by 8; $\frac{7}{10}$ by 12; $\frac{7}{20}$ by 9.

10. Multiply $\frac{21}{25}$ by 25; $\frac{11}{15}$ by 15; $\frac{13}{15}$ by 30; $\frac{15}{32}$ by 16.

11. How much is 5 times $6\frac{2}{3}$? 7 times $8\frac{1}{2}$?

SUGGESTION.—Multiply the fraction and then the integer, and add the products.

12. How much is 8 times $6\frac{1}{4}$? 6 times $16\frac{2}{3}$?

13. Multiply $18\frac{3}{4}$ by 12; $66\frac{2}{3}$ by 15; $80\frac{3}{5}$ by 20.

14. Multiply $33\frac{1}{3}$ by 15; $30\frac{3}{9}$ by 10; $64\frac{3}{4}$ by 16.

15. Multiply $40\frac{2}{5}$ by 30; $56\frac{7}{8}$ by 24; $90\frac{5}{6}$ by 60.

FRACTIONS MULTIPLIED BY FRACTIONS.

1. How much is $\frac{1}{2}$ of $\frac{1}{4}$ of a melon?

2. If a third of a melon be cut into two equal pieces, what part of the melon will each piece be?

3. What is $\frac{1}{2}$ of $\frac{1}{3}$ of a melon? $\frac{1}{2}$ of $\frac{1}{4}$ of a melon?

4. What is $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{3}$?

5. What is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{7}$?

6. What is $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{9}$? $\frac{1}{5}$ of $\frac{1}{6}$?

7. A girl, having $\frac{3}{4}$ of an orange, divided it equally between her two brothers: what part of the orange did each receive?

SUGGESTION.—Divide each fourth into two equal pieces, and then give 3 pieces to each. $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, and $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

8. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{2}$ of $\frac{5}{4}$? $\frac{1}{2}$ of $\frac{7}{4}$?

9. What is $\frac{1}{3}$ of $\frac{1}{8}$? $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{1}{3}$ of $\frac{5}{8}$? $\frac{1}{3}$ of $\frac{7}{8}$?

10. What is $\frac{1}{5}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{3}{7}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{3}{4}$ of $\frac{3}{5}$?

SOLUTION.—$\frac{1}{4}$ of $\frac{3}{5}$ is $\frac{3}{20}$, and $\frac{3}{4}$ of $\frac{3}{5}$ is 3 times $\frac{3}{20}$, which is $\frac{9}{20}$.

11. What is $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{3}{8}$ of $\frac{2}{7}$? $\frac{3}{5}$ of $\frac{5}{6}$?

12. How much is $\frac{1}{4}$ times $\frac{2}{3}$? $\frac{2}{4}$ times $\frac{2}{3}$? $\frac{3}{4}$ times $\frac{2}{3}$?

Solution.—$\frac{1}{4}$ times $\frac{2}{3}$ is $\frac{1}{4}$ of $\frac{2}{3}$, which is $\frac{2}{12}$, or $\frac{1}{6}$.

13. How much is $\frac{1}{3}$ times $\frac{3}{4}$? $\frac{2}{3}$ times $\frac{3}{4}$? $\frac{2}{3}$ times $\frac{5}{6}$?

WRITTEN EXERCISES.

14. Multiply $\frac{4}{5}$ by $\frac{3}{4}$.

PROCESS: $\frac{4}{5}\times\frac{3}{4}=\frac{4\times 3}{5\times 4}=\frac{12}{20}=\frac{3}{5}$, *Ans.*

15. Solve examples 11, 12, and 13 above on paper.

16. Multiply $\frac{5}{8}$ by $\frac{2}{5}$.
17. Multiply $\frac{4}{7}$ by $\frac{7}{8}$.
18. Multiply $\frac{2}{15}$ by $\frac{5}{8}$.
19. Multiply $\frac{5}{6}$ by $\frac{6}{13}$.
20. Multiply $\frac{7}{10}$ by $\frac{7}{12}$.

21. $\frac{13}{4}$ by $\frac{2}{13}$.
22. $\frac{10}{17}$ by $\frac{17}{15}$.
23. $\frac{5}{9}$ by $\frac{9}{20}$.
24. $\frac{9}{25}$ by $\frac{5}{6}$.
25. $2\frac{1}{2}$ by $\frac{5}{6}$.

26. $\frac{7}{12}$ by $\frac{6}{10}$.
27. $\frac{3}{5}$ by $\frac{7}{9}$.
28. $1\frac{2}{3}$ by $\frac{2}{5}$.
29. $\frac{7}{10}$ by $3\frac{1}{3}$.
30. $2\frac{1}{2}$ by $\frac{4}{15}$.

31. Multiply the multiplicand in each of the above examples from 16 to 25 by $\frac{2}{3}$; from 26 to 30 by $\frac{3}{5}$.

32. Multiply (1) 16 by $\frac{3}{4}$; (2) $\frac{5}{9}$ by 6; (3) $\frac{7}{12}$ by $\frac{3}{7}$.

PROCESS: (1) $\frac{16}{1}\times\frac{3}{4}=\frac{48}{4}=12$; (2) $\frac{5}{9}\times\frac{6}{1}=\frac{30}{9}=3\frac{1}{3}$; (3) $\frac{7}{12}\times\frac{3}{7}=\frac{21}{84}=\frac{1}{4}$.

Note.—It is thus seen that the three processes in multiplication may be reduced to one by writing the integer in the form of a fraction, as above.

33. Multiply 27 by $\frac{5}{9}$; $\frac{4}{7}$ by 20; $\frac{5}{11}$ by $\frac{7}{10}$; $2\frac{1}{2}$ by $\frac{3}{8}$.
34. Multiply 63 by $\frac{7}{9}$; $\frac{7}{8}$ by 12; $\frac{5}{8}$ by $\frac{3}{5}$; $3\frac{1}{3}$ by $\frac{2}{5}$.
35. Multiply 24 by $\frac{7}{8}$; $\frac{5}{6}$ by 10; $\frac{4}{5}$ by $\frac{5}{6}$; $\frac{3}{4}$ by $3\frac{2}{3}$.
36. Multiply 56 by $\frac{7}{8}$; $\frac{5}{6}$ by 6; $\frac{7}{12}$ by $\frac{4}{7}$; $5\frac{1}{2}$ by 6.
37. Multiply $\frac{3}{5}$ by $\frac{3}{4}$; $2\frac{1}{2}$ by $\frac{2}{5}$; 45 by $\frac{3}{8}$; 80 by $\frac{7}{16}$.

To multiply one fraction by another:—

Rule.—*Multiply the numerators together and also the denominators, and reduce the resulting fraction to its lowest terms.*

Note.—The process may often be shortened by cancellation, but this should not be used until the full process is mastered.

PROBLEMS.

N. B.—These problems are to be solved orally and then on paper.

1. What will 6 pounds of raisins cost at $\$\frac{1}{5}$ a pound?

2. What will $\frac{3}{4}$ of a yard of silk cost at $\$\frac{5}{6}$ a yard?

3. At $15 a ton, what will $\frac{3}{4}$ of a ton of hay cost?

4. At $4 a yard, what will $3\frac{1}{2}$ yards of cloth cost?

5. A man bought $\frac{5}{8}$ of an acre of land and then sold $\frac{1}{2}$ of it: what part of an acre did he sell?

6. Harry is 16 years old and Henry is $\frac{3}{4}$ as old as Harry: how old is Henry?

7. A horse eats $\frac{3}{8}$ of a bushel of oats in a day: how much will he eat in 6 days? In 10 days?

8. At $12\frac{1}{2}$ cents a pound, what will 6 pounds of cheese cost? 8 pounds? 10 pounds? 12 pounds?

9. A man earns $120 a month and pays $\frac{1}{5}$ of it for house-rent: how much does he pay for house-rent?

10. A man pays $24 a month for house-rent, which is $\frac{2}{5}$ of what he earns: how much does he earn?

DIVISION OF FRACTIONS.

FRACTIONS DIVIDED BY INTEGERS.

1. A father divided $\frac{3}{4}$ of a melon equally among 3 boys: what part of the melon did each receive?

SOLUTION.—If 3 boys received $\frac{3}{4}$ of a melon, each received $\frac{1}{3}$ of $\frac{3}{4}$ of the melon, which is $\frac{1}{4}$.

2. How much is $\frac{6}{5}$ divided by 3?

SOLUTION.—$\frac{6}{5} \div 3$ is $\frac{1}{3}$ of $\frac{6}{5}$, which is $\frac{2}{5}$.

3. How much is $\frac{8}{9}$ divided by 4? $\frac{21}{4}$ by 7? $\frac{10}{11}$ by 5?

4. Divide $\frac{3}{4}$ by 5; $\frac{10}{3}$ by 5; $\frac{3}{5}$ by 6; $\frac{4}{5}$ by 8; $\frac{7}{8}$ by 4. $\frac{4}{9}$ by 6; $\frac{6}{7}$ by 3; $\frac{6}{7}$ by 9; $\frac{14}{15}$ by 7.

5. Divide $\frac{7}{8}$ by 5; $\frac{15}{7}$ by 6; $\frac{10}{11}$ by 4; $12\frac{1}{2}$ by 4.

Solutions.—$\frac{10}{11} \div 4$ is $\frac{1}{4}$ of $\frac{10}{11}$, which is $\frac{10}{44} = \frac{5}{22}$. $12\frac{1}{2} \div 4$ is $\frac{1}{4}$ of $12\frac{1}{2}$; $\frac{1}{4}$ of 12 is 3 and $\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{8}$: $\frac{1}{4}$ of $12\frac{1}{2}$ is $3\frac{1}{8}$.

6. Divide $18\frac{1}{3}$ by 6; $20\frac{2}{3}$ by 5; $30\frac{1}{4}$ by 3; $13\frac{1}{3}$ by 4; $26\frac{1}{3}$ by 5; $33\frac{1}{3}$ by 4; $35\frac{1}{2}$ by 8; $42\frac{1}{2}$ by 5.

WRITTEN EXERCISES.

7. Divide $\frac{15}{11}$ by 3; $\frac{7}{20}$ by 4.

PROCESS.

(1) $\frac{15}{11} \div 3 = \frac{1}{3}$ of $\frac{15}{11} = \frac{5}{11}$, *Ans.*

(2) $\frac{7}{20} \div 4 = \frac{1}{4}$ of $\frac{7}{20} = \frac{7}{80}$, *Ans.*

It is thus seen that a fraction is divided by dividing its numerator or multiplying its denominator.

8. Divide $\frac{14}{11}$ by 7.
9. Divide $\frac{12}{7}$ by 6.
10. Divide $\frac{7}{5}$ by 5.
11. Divide $\frac{9}{13}$ by 3.
12. Divide $\frac{12}{13}$ by 4.
13. Divide $\frac{10}{9}$ by 5.
14. $\frac{16}{15}$ by 8.
15. $\frac{20}{21}$ by 10.
16. $\frac{21}{25}$ by 7.
17. $\frac{15}{8}$ by 6.
18. $\frac{8}{5}$ by 6.
19. $\frac{7}{9}$ by 9.
20. $16\frac{2}{3}$ by 2.
21. $25\frac{1}{7}$ by 8.
22. $46\frac{1}{4}$ by 5.
23. $66\frac{2}{3}$ by 8.
24. $33\frac{1}{3}$ by 8.
25. $62\frac{1}{2}$ by 5.

FRACTIONS DIVIDED BY FRACTIONS.

1. If $\frac{2}{3}$ of a yard of cloth will make a vest, how many vests will 4 yards make?

Solution.—4 yards contain $\frac{12}{3}$ of a yard; and $\frac{2}{3}$ of a yard is contained in $\frac{12}{3}$ of a yard 6 times. 4 yards will make 6 vests.

2. How many times is $\$\frac{3}{4}$ contained in \$3? In \$9?
3. How many times is $\frac{3}{5}$ contained in 6? $\frac{4}{5}$ in 8?
4. How many times $\frac{2}{8}$ in 12? $\frac{3}{4}$ in 12? $\frac{3}{5}$ in 12?
5. Divide $\frac{6}{8}$ of a quart by $\frac{2}{8}$ of a quart; by $\frac{3}{8}$ of a quart.
6. Divide $\frac{12}{5}$ by $\frac{3}{5}$; $\frac{12}{5}$ by $\frac{4}{5}$; $\frac{16}{7}$ by $\frac{4}{7}$; $\frac{15}{8}$ by $\frac{5}{8}$.
7. Divide $\frac{1}{2}$ of a peck by $\frac{1}{4}$ of a peck.

Suggestion.—Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to like fractions.

8. Divide $\frac{1}{2}$ by $\frac{1}{3}$; $\frac{3}{4}$ by $\frac{2}{3}$; $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{3}{4}$ by $\frac{3}{5}$.

WRITTEN EXERCISES.

9. Divide $\frac{5}{7}$ by $\frac{2}{3}$.

Reduce the fractions to like fractions, and then divide the numerators. 15 twenty-firsts ÷ 14 twenty-firsts = 15 ÷ 14 = $\frac{15}{14}$.

PROCESS.

$\frac{5}{7} = \frac{15}{21}$; $\frac{2}{3} = \frac{14}{21}$;

$\frac{15}{21} \div \frac{14}{21} = 15 \div 14 = 1\frac{1}{14}$, *Ans.*

10. Divide $\frac{5}{6}$ by $\frac{5}{8}$.
11. Divide $\frac{6}{7}$ by $\frac{3}{4}$.
12. Divide $\frac{5}{8}$ by $\frac{5}{6}$.
13. Divide $\frac{7}{8}$ by $\frac{3}{4}$.
14. Divide $\frac{3}{5}$ by $\frac{1}{2}$.
15. Divide $\frac{3}{12}$ by $\frac{5}{8}$.
16. Divide $\frac{6}{7}$ by $\frac{2}{3}$.
17. Divide $\frac{5}{9}$ by $\frac{2}{3}$.
18. $2\frac{1}{2}$ by $\frac{2}{3}$.
19. $3\frac{1}{3}$ by $\frac{3}{5}$.
20. $\frac{3}{5}$ by $2\frac{1}{2}$.
21. $\frac{5}{8}$ by $\frac{3}{4}$.
22. $\frac{3}{4}$ by $\frac{3}{8}$.
23. $\frac{3}{5}$ by $\frac{7}{8}$.
24. $\frac{7}{8}$ by $\frac{3}{5}$.
25. $\frac{2}{3}$ by $\frac{5}{6}$.
26. $\frac{5}{6}$ by $\frac{2}{3}$.
27. 5 by $\frac{5}{7}$.
28. $\frac{3}{5}$ by $\frac{7}{9}$.
29. 8 by $\frac{3}{4}$.
30. $\frac{3}{4}$ by $\frac{5}{12}$.
31. 8 by $\frac{5}{6}$.
32. 9 by $\frac{6}{7}$.
33. $6\frac{2}{3}$ by $\frac{5}{6}$.

34. Divide $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, $\frac{7}{12}$, $2\frac{1}{2}$, and 5 each by $\frac{3}{8}$.

To divide a fraction by a fraction : —

RULE.—*Reduce the fractions to like fractions, and then divide the numerator of the dividend by the numerator of the divisor.*

NOTE.—The method involving the inversion of the terms of the divisor need not be taught to young pupils.

QUESTIONS FOR REVIEW.

How are like fractions added? How are they subtracted?

When fractions are not like fractions, how are they added? How are they subtracted?

How may mixed numbers be added? How may they be subtracted?

In what two ways may a fraction be multiplied by an integer? In what two ways may a fraction be divided by an integer?

How may a fraction be divided by a fraction? How may 4 be divided by $\frac{2}{3}$? How may $2\frac{1}{3}$ be divided by $\frac{2}{3}$?

MISCELLANEOUS PROBLEMS.

1. How much is $\frac{5}{8}$ of 56? $\frac{5}{7}$ of 50?

2. How much is $\frac{1}{3}$ of $22\frac{1}{2}$? $\frac{1}{5}$ of $62\frac{1}{2}$?

3. $16\frac{2}{3}$ is $\frac{1}{6}$ of what number?

4. 27 is $\frac{3}{7}$ of what number?

5. 32 is $\frac{5}{6}$ of what number?

6. 60 feet are $\frac{5}{8}$ of how many feet?

7. Clara is 12 years old, and her age is $\frac{2}{9}$ of her father's age: how old is her father?

8. Edward is 21 years old, and his age is $\frac{3}{7}$ of his mother's age: how old is his mother?

9. A man spent $21, which was $\frac{3}{5}$ of his money: how much money had he at first?

10. A boy gave 24 cents for a slate, which was $\frac{4}{7}$ of his money: how much money did he have?

11. A man pays $25 a month for house-rent, which is $\frac{5}{12}$ of his monthly salary: what is his salary?

12. A farmer sold a cow for $45, which was $\frac{5}{4}$ of what he paid for her: what was the cost of the cow?

13. A farm contains 90 acres, and $\frac{4}{15}$ of it is woodland: how many acres are woodland?

14. $\frac{1}{4}$ of a farm is in wheat, $\frac{1}{3}$ of it in corn, and the rest in grass: what part of the farm is in grass?

15. A boy spent $\frac{1}{3}$ of his money for a sled, and $\frac{2}{5}$ of it for a pair of skates: what part had he left?

16. John earned a dollar and then paid $\frac{3}{8}$ of a dollar for a knife and $\frac{1}{4}$ of a dollar for a ball: what part of a dollar had he left?

17. Jane having a quart of plums, gave $\frac{1}{3}$ of a quart to her brother and $\frac{1}{4}$ of a quart to her sister: what part of a quart had she left?

18. A farmer bought 2 bushels of clover-seed, and then sold $\frac{2}{5}$ of a bushel to one neighbor and $\frac{3}{4}$ of a bushel to another: what part of a bushel had he left?

19. A student spends $\frac{2}{5}$ of his time in study, $\frac{1}{10}$ of it in labor, and $\frac{1}{3}$ of it in sleep: what part of it is left?

20. One sixth of a pole is in the ground, $\frac{2}{5}$ of it in water, and the rest in the air: what part is in the air?

21. A man sold $\frac{2}{5}$ of his farm to one neighbor and $\frac{3}{8}$ of it to another: what part of the farm has he left?

22. A boy having $\frac{3}{4}$ of a melon, gave $\frac{1}{2}$ of it to a playmate: what part did the playmate receive?

23. A woman divided $\frac{4}{5}$ of a pound of raisins equally among 3 children: how much did each receive?

24. If 6 men can build $\frac{7}{8}$ of a wall in a day, what part of the wall can one man build?

25. If 5 pounds of cheese cost $\$\frac{3}{4}$, what will 1 pound cost? What will 6 pounds cost?

26. A grocer put $16\frac{1}{2}$ pounds of sugar into 3 equal parcels: how much sugar was put into each parcel?

27. If a basket hold $\frac{2}{5}$ of a bushel, how many baskets will hold 4 bushels?

28. If a pound of butter cost $\$\frac{1}{4}$, how many pounds can be bought for $\$2\frac{1}{2}$?

29. If $\frac{2}{5}$ of a yard of cloth will make a vest, how many vests will $1\frac{3}{5}$ yards make?

30. A man owning $\frac{5}{8}$ of a factory sold $\frac{2}{5}$ of his share: what part of the factory did he sell? What part did he still own?

WRITTEN PROBLEMS.

31. Reduce $18\frac{3}{4}$ to an improper fraction.
32. Reduce $\frac{113}{20}$ to a mixed number.
33. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to like fractions.
34. Add $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{4}$, and $\frac{5}{12}$.
35. Add $28\frac{3}{5}$, $40\frac{1}{2}$, $63\frac{2}{3}$, and $19\frac{3}{10}$.
36. From $28\frac{2}{5}$ take $16\frac{2}{3}$; from $30\frac{1}{4}$ take $12\frac{1}{2}$.
37. Multiply $\frac{5}{8}$ by 7; 13 by $\frac{3}{5}$; $\frac{3}{4}$ by $\frac{6}{7}$.
38. Multiply $137\frac{2}{3}$ by 15; 256 by $21\frac{5}{8}$.

39. Divide 12 by $\frac{2}{3}$; $\frac{2}{3}$ by 12; $\frac{3}{5}$ by $\frac{6}{7}$.

40. Divide $2\frac{1}{2}$ by $\frac{2}{3}$; $\frac{5}{8}$ by $1\frac{1}{4}$; $1\frac{1}{2}$ by $1\frac{1}{3}$.

41. $\frac{7}{8}+\frac{2}{3}=$ what? $\frac{7}{8}-\frac{2}{3}$? $\frac{7}{8}\times\frac{2}{3}$? $\frac{7}{8}\div\frac{2}{3}$?

42. There are 5280 feet in a mile: how many feet in $\frac{9}{40}$ of a mile? In $\frac{5}{16}$ of a mile?

43. A vessel is worth $6000, and the cargo is worth $\frac{5}{8}$ as much as the vessel: what is the value of the cargo?

44. There are $16\frac{1}{2}$ feet in a rod: how many feet in 66 rods? In 120 rods? In 160 rods? In 320 rods?

45. There are $5\frac{1}{2}$ yards in a rod: how many yards in 66 rods? In 80 rods? In 200 rods?

46. At $\$6\frac{1}{4}$ a barrel, what will 84 barrels of flour cost?

47. If $\frac{4}{7}$ of a ship is worth $12000, what is the whole ship worth?

48. A man sold $\frac{3}{7}$ of a farm for $1500: at this rate, what was the value of the farm?

49. A farmer had two fields of wheat; the first yielded 840 bushels, and the second yielded $\frac{5}{12}$ as much as the first: how many bushels did the second field yield?

50. Two men bought a newspaper for $3600, one paying $\frac{2}{3}$ and the other $\frac{1}{3}$: how much did each pay?

51. A farmer bought $240\frac{5}{8}$ acres of land, and sold $90\frac{2}{3}$ acres and $75\frac{1}{4}$ acres: how many acres had he left?

52. From a piece of broadcloth containing $20\frac{3}{4}$ yards, a merchant sold $5\frac{1}{2}$ yards, $4\frac{1}{3}$ yards, and $8\frac{1}{4}$ yards: how many yards were left?

53. A man bought $\frac{3}{4}$ of a section of land and then sold $\frac{1}{4}$ of what he bought: what part of a section did he sell? What part had he left?

54. A pedestrian walked $\frac{4}{15}$ of his journey the first day, $\frac{5}{12}$ of it the next day, and completed it the third day: what part of the journey did he travel the third day?

55. A man sold $\frac{1}{4}$ of his farm to A, $\frac{1}{3}$ of it to B, and the rest to C: what part of the farm did he sell to C?

56. A man owning $\frac{3}{5}$ of a ship sells $\frac{2}{3}$ of his share for $4400: at this rate, what is the value of the ship?

57. What is the weight of 56 sacks of salt if each weighs $12\frac{3}{4}$ pounds?

58. If a barrel holds $2\frac{3}{4}$ bushels of apples, how many barrels will hold 132 bushels?

59. A man earns \$$2\frac{1}{2}$ a day: in how many days will he earn \$150?

60. If a man's steps are $2\frac{1}{2}$ feet each, how many steps will he take in walking 250 feet?

61. Two brothers paid \$4250 for a mill, one paying $\frac{2}{5}$ of the cost and the other $\frac{3}{5}$: how much did each pay?

62. A man bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of it to each of his two sons, and the rest to his daughter: what part did the daughter receive?

63. A primary school enrolls 240 pupils; $\frac{2}{5}$ of the pupils are in the first grade, $\frac{5}{12}$ in the second grade, and the others in the third grade: how many pupils in each grade?

64. A fruit-dealer bought 80 barrels of apples, each containing $2\frac{3}{4}$ bushels, at \$$2\frac{1}{2}$ per barrel, and then sold the apples at \$$1\frac{1}{5}$ per bushel: how much did he gain?

65. A man who owned $\frac{3}{4}$ of a newspaper sold $\frac{1}{3}$ of his share to his brother and $\frac{1}{4}$ of it to his sister: what part of the paper did each buy? What part did the man then own?

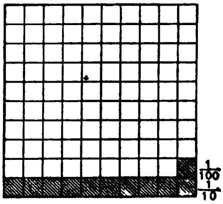
66. A man divided 180 acres of land among three sons, the eldest receiving $\frac{2}{5}$ of it, the second son $\frac{1}{3}$ of it, and the youngest son what remained: how many acres did each son receive?

67. Two men start from the same place and walk in opposite directions, one $2\frac{3}{4}$ miles an hour and the other $3\frac{1}{2}$ miles an hour: how far apart will they be in 1 hour? In 4 hours? In 10 hours?

68. A grocer bought 4 crocks of butter, weighing respectively $31\frac{3}{8}$ pounds, $28\frac{3}{4}$ pounds, $27\frac{5}{8}$ pounds, and $30\frac{3}{4}$ pounds, and the weight of the empty crocks was $22\frac{1}{2}$ pounds: what was the weight of the butter? What was its cost at $33\frac{1}{3}$ cents a pound?

DECIMAL FRACTIONS.

NUMERATION AND NOTATION.



1. If a square be divided into ten equal parts, what is one part?

2. If a tenth of a square be divided into ten equal parts, what part of the square is one of these parts? How much is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If a hundredth of a square be divided into ten equal parts, what part of the square is one of these parts? How much is $\frac{1}{10}$ of $\frac{1}{100}$?

4. How many tenths in a unit? How many hundredths in one tenth? How many thousandths in one hundredth?

The division of a unit into tenths, tenths into hundredths, hundredths into thousandths, etc., is called a *decimal* division, and the fractions arising from such a decimal division of the unit is called a *decimal fraction.* Thus, five tenths, fifteen hundredths, forty-five thousandths, etc., are decimal fractions.

Decimal fractions may be expressed by figures by writing the numerator over the denominator; as, $\frac{5}{10}$, $\frac{15}{100}$, $\frac{245}{1000}$, etc. When a decimal fraction is thus expressed it is written in the *common form, i. e.,* the form common to all fractions.

Since 10 thousandths equal 1 hundredth, 10 hundredths 1 tenth, and 10 tenths 1 unit, decimal fractions may also be expressed, like integers, on a *scale of ten,* called the *decimal scale.* This is done by writing the tenths in the first order at the right of units, the hundredths in the second order, the thousandths in the third order, etc., and placing a period, called the *decimal point,* at the left of the order of tenths. Thus, $\frac{5}{10}$ may be written .5; $\frac{15}{100}$, .15; $\frac{45}{1000}$, .045, etc. This is called the *decimal form* of writing decimal fractions.

5. How many tenths in .3?. In .6? .7? .8? .9?

6. How many hundredths in .02? In .04? .07?

7. How many tenths and hundredths in .24? .36?

8. How many thousandths in .005? .008? .006?

9. How many tenths, hundredths, and thousandths in .356? In .523? .603? .041? .406? .007?

The decimal .3 is read *three tenths;* .24 is read *twenty-four hundredths;* and .205 is read *two hundred five thousandths.*

10. Read .6; .7; .8; .25; .36; .45; .125; .244.

11. Read .06; .008; .045; .04; .014; .503.

NOTE.—A decimal fraction expressed in the decimal form is read precisely as it would be if written in the common form.

12. Read $\frac{9}{10}$, .9; $\frac{63}{100}$, .63; $\frac{503}{1000}$, .503; $\frac{24}{1000}$, .024.

When decimal fractions are written in the decimal form they are expressed *decimally,* and they are then called briefly *decimals.*

Write decimally the following fractions:—

13.	**14.**	**15.**	**16.**	**17.**
$\frac{3}{10}$	$\frac{25}{100}$	$\frac{225}{1000}$	$\frac{6}{10}$	$2\frac{3}{10}$
$\frac{5}{10}$	$\frac{6}{100}$	$\frac{808}{100}$	$\frac{6}{100}$	$24\frac{15}{100}$
$\frac{7}{10}$	$\frac{81}{100}$	$\frac{27}{1000}$	$\frac{6}{1000}$	$65\frac{35}{1000}$

18.	**19.**	**20.**
25 hundredths.	$26\frac{1}{3}$ hundredths.	18 hundredths.
7 hundredths.	$6\frac{1}{3}$ thousandths.	204 thousandths.
205 thousandths.	28 thousandths.	50 thousandths.
68 hundredths.	624 thousandths.	300 thousandths.
68 thousandths.	97 hundredths.	40 hundredths.
405 thousandths.	106 thousandths.	405 thousandths.

The fourth decimal order is *ten-thousandths;* the fifth, *hundred-thousandths;* and the sixth, *millionths.*

The decimal .0205 is read two hundred five *ten-thousandths;* .02005 is read two thousand five *hundred-thousandths;* .000024 is read twenty-four *millionths.* The name of a decimal is *the name of its right-hand order.*

Copy and read the following decimals:—

21.	**22.**	**23.**	**24.**
.445	.0304	.3256	.00078
.0445	.0055	.03256	.000078
.706	.2008	.4048	.00011
.0706	.3076	.04048	.000101
.0063	.0376	.30003	.000036

Write decimally the following fractions:—

25.	**26.**	**27.**	**28.**
$\frac{2124}{10000}$	$\frac{3406}{100000}$	$\frac{405}{1000}$	$\frac{34}{100}$
$\frac{304}{10000}$	$\frac{236}{100000}$	$\frac{4005}{100000}$	$\frac{304}{1000}$
$\frac{24}{10000}$	$\frac{34}{100000}$	$\frac{45}{10000}$	$\frac{3004}{100000}$
$\frac{8}{10000}$	$\frac{7}{100000}$	$\frac{652}{100000}$	$\frac{16}{1000000}$

29. Four hundred twenty-two thousandths.

30. Seven hundred five ten-thousandths.

31. Fifteen millionths.

32. Four hundred fourteen hundred-thousandths.

An integer and a decimal united as one number is a *mixed decimal.* Thus, 5.4 and 25.24 are mixed decimals.

The orders on the left of the decimal point are *integral* orders, and those on the right are *decimal* orders. Decimal orders are also called *decimal places.*

When a mixed decimal is expressed in words the integer and the decimal are connected by *and.* Thus, 45.14 is read *forty-five units and fourteen hundredths*, or, omitting "units," *forty-five and fourteen hundredths.*

33. Read 12.6; 15.045; 25.03; 7.009

34. Read 27.305; 6.0346; 504.35; 2.003

35. Read 400.04; 5000.004; 400.0012; 300.006
36. Write decimally $25\frac{14}{100}$; $6\frac{8}{100}$; $46\frac{33}{1000}$; $600\frac{3}{100}$.
37. Forty-five units and fifty-two hundredths.
38. Forty units and forty-five thousandths.
39. Two hundred units and seventy-nine millionths.
40. Two hundred seventy-nine millionths.
41. Five thousand units and sixty-six millionths.
42. Five thousand sixty-six millionths.
43. Fourteen units and fourteen hundred-thousandths.
44. One thousand eighty-seven millionths.
45. Eight hundred forty-two ten thousandths.

A Decimal Fraction is one or more of the decimal parts of a unit.

The *decimal parts* of a unit result from its division into tenths, tenths into hundredths, hundredths into thousandths, etc., called the *decimal* division.

The denominators of decimal fractions are 10, 100, 1000, 10000, etc. When a decimal fraction is expressed in the decimal form, the denominator is not written.

The successive decimal orders decrease in value from left to right and increase from right to left in the same manner as integral orders.

The following table gives the names of six integral and six decimal orders: —

Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
6	5	4	3	2	1	.	1	2	3	4	5	6
Integral Orders.							**Decimal Orders.**					

REDUCTION OF DECIMALS.

DECIMALS REDUCED TO COMMON FRACTIONS.

1. Reduce .75 to a common fraction in lowest terms.

Write .75 in the common form ($\frac{75}{100}$) and then reduce the fraction to its lowest terms.

PROCESS.

$.75 = \frac{75}{100} = \frac{3}{4}$, *Ans.*

Reduce to a common fraction in lowest terms:—

2. .25
3. .75
4. .125
5. .625
6. .375
7. .025
8. .035
9. .075
10. .024
11. .225
12. .275
13. .0025
14. .0125
15. .0150
16. .032
17. .013
18. 6.25
19. 13.75
20. 4.125
21. 25.375
22. 2.075
23. 216.5
24. 24.35
25. 30.12

COMMON FRACTIONS REDUCED TO DECIMALS.

26. How many tenths in $\frac{1}{2}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?
27. How many hundredths in $\frac{1}{10}$? $\frac{3}{10}$? $\frac{1}{25}$? $\frac{7}{25}$? $\frac{12}{25}$?
28. Reduce $\frac{8}{25}$ to a decimal.

ORAL PROCESS.

$\frac{8}{25}$ is $\frac{1}{25}$ of 8, or $\frac{1}{25}$ of 8.00
which is .32 $\frac{8}{25} = .32$

The result contains as many decimal places as there are annexed decimal ciphers.

WRITTEN PROCESS.

```
25)8.00(.32
   7 5
   ───
     50
     50
     ──
```

29. Reduce $\frac{5}{8}$ to a decimal. *Ans.* .625

Reduce the following common fractions to decimals:—

30. $\frac{1}{2}$
31. $\frac{1}{5}$
32. $\frac{3}{4}$
33. $\frac{3}{5}$
34. $\frac{2}{8}$
35. $\frac{5}{8}$
36. $\frac{1}{8}$
37. $\frac{3}{25}$
38. $\frac{3}{40}$
39. $\frac{5}{20}$
40. $\frac{32}{125}$
41. $\frac{3}{50}$
42. $12\frac{1}{2}$
43. $56\frac{1}{4}$
44. $30\frac{3}{4}$
45. $7\frac{3}{5}$
46. $6\frac{1}{2}$
47. $12\frac{1}{4}$
48. $45\frac{2}{5}$
49. $24\frac{5}{8}$

ADDITION OF DECIMALS.

1. What is the sum of 5 tenths and 4 tenths? 6 tenths and 3 tenths? 6 tenths and 9 tenths?

Solution.—6 tenths and 9 tenths are 15 tenths, equal to 1 unit and 5 tenths.

2. What is the sum of .4 and .8? .5 and .9? .36 and .08? .24 and .08?

WRITTEN EXERCISES.

3. What is the sum of 24.6, 307.08, 93.609, 400.06, .456?

WRITTEN PROCESS.

```
 24.6
307.08
 93.609
400.06
   .456
825.805, Ans.
```

Since only like orders can be added, write the figures of the same order in the same column.

Begin at the right and add as in integers, and place the decimal point at the left of the tenths' order in the sum.

4. What is the sum of .375, .806, .095, .45, and .9?
5. What is the sum of .4506, .709, and 27.508?
6. Add 15.34, 6.078, 60.804, 99.875, and .674
7. Add $21.94, $87.075, $9.858, and $807.62½.
8. Add 47.6 miles, 19.48 miles, 34.75 miles, 19.08 miles, 108.6 miles, and 76.625 miles.
9. Add thirty-nine hundredths; six hundred eight ten-thousandths; and eighty-seven thousandths.
10. Add forty-seven thousandths; forty-seven hundredths; sixteen ten-thousandths; and six hundredths.
11. Add four hundred nine thousandths; fifty-six ten-thousandths; one thousand eight hundred six hundred-thousandths; seven hundred fifty-four ten-thousandths; and eight hundred eighty-eight hundred-thousandths.
12. Add seven thousand seven hundred seven hundred-thousandths; sixty-six thousand eight hundred eight

millionths; seven hundred nine thousand forty-eight millionths; and eight hundred nine millionths.

13. What is the weight of five bars of lead, which weigh respectively 4.75 pounds, 3.6 pounds, 10.35 pounds, 6.375 pounds, and 5.8 pounds?

SUBTRACTION OF DECIMALS.

1. From 56.403 take 18.6

PROCESS:
56.403
18.6
37.803, *Ans.*

2. From 56.6 take 18.403

PROCESS:
56.600
18.403
38.197, *Ans.*

Since only like units can be subtracted, write the figures of the same order in the same column.

Begin at the right and subtract as in integers, and place the decimal point at the left of the tenths' order in the difference.

3. From 45.3 take 28.756; from 40.74 take 19.086

4. From .0407 take .0075; from .081 take .0076

5. From seven hundredths take twelve thousandths.

6. From 47.065+36.87 take 9.08+43.375

7. From 5.25+.087 take 5.163

8. From eight tenths take eight ten-thousandths.

9. From fifteen hundredths take six thousandths.

10. From the sum of twenty-five thousandths and forty-six hundredths take two hundred six ten-thousandths.

11. From seven hundred seventy-seven ten-thousandths take the sum of sixty-seven ten-thousandths and four hundred six hundred-thousandths.

12. A man bought 45.75 pounds of flax-seed and sold 26.62½ pounds: how many pounds had he left?

To add or subtract decimals:—

RULE.—*Write units of the same order in the same column, add or subtract as in integers, and place the decimal point at the left of the order of tenths.*

MULTIPLICATION OF DECIMALS.

1. How much is 3 times 2 tenths? 3 times $\frac{5}{10}$?

2. How much is 4 times 3 tenths? 4 times .4?

3. How much is 5 times $\frac{3}{100}$? 5 times .05?

4. How much is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{1}{10}$ of $\frac{1}{1000}$?

5. How much is $\frac{1}{100} \times \frac{1}{10}$? $\frac{1}{100} \times \frac{1}{100}$? $\frac{1}{100} \times \frac{1}{1000}$?

6. What is the product when tenths are multiplied by tenths? Tenths by hundredths? Hundredths by hundredths? Tenths by thousandths?

WRITTEN EXERCISES.

7. Multiply .435 by .65

PROCESS.

$$\begin{array}{r} .435 \\ \underline{.65} \\ 2175 \\ \underline{2610} \\ .28275, \textit{ Ans.} \end{array}$$

Since thousandths multiplied by hundredths produce *hundred-thousandths* ($\frac{435}{1000} \times \frac{65}{100} = \frac{28275}{10000}$), the product of .425 and .65 contains *five* decimal orders, or as many as in both multiplicand and multiplier.

8. Multiply .015 by .03; by .024

9. Multiply .0046 by 6.4; by .25

10. Multiply .047 by 2.4; by 24; by .24

11. Multiply .625 by 4.5; by .45; by .045

12. Multiply .286 by .064; by .64; by 6.4

13. Multiply .256 by 5.4; 2.56 by .54; 25.6 by .054

14. Multiply 64.5 by .75; 6.45 by 7.5; .645 by .075

15. Multiply 86.4 by 8.5; by .85

16. Multiply .347 by .73; by 7.3

17. Multiply .48 by 3.6; by 36.

18. Multiply .067 by .65; by 6.5

19. Multiply 3.42 by 5.4; by .054

20. Multiply 47.5 by .034; by 3.4

21. Multiply 492 by 30.6; by 3.06

22. Multiply 650 by 2.4; by .0024

23. 30.3 by .044

24. .008 by .007

25. .075 by $.48\frac{1}{3}$

26. 2.42 by 50.9

27. .0075 by 2.8

28. 43.6 by .073

29. .024 by $.06\frac{1}{2}$

30. Multiply the multiplicand in each of the above problems, from 16 to 29, by .035; by 3.5

31. Multiply 4.354 by 10; by 100; by 1000.

PROCESS.

$4.354 \times 10 = 43.54$, *Ans.*
$4.354 \times 100 = 435.4$, *Ans.*
$4.354 \times 1000 = 4354.$, *Ans.*

The removal of the decimal point one place to the right multiplies 4.354 by 10; two places to the right by 100; three places to the right by 1000, etc.

To Teachers.—Show that the removal of a decimal figure one place to the left multiplies its value by 10.

32. Multiply 4.085 by 100; by 1000; by 10.

33. Multiply 3.0048 by 100; by 1000; by 10000.

34. Multiply 41.265 by 100; by 1000; by 10000.

To multiply one decimal by another:—

Rule.—*Multiply as in the multiplication of integers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.*

Note.—If there be not enough decimal figures in the product, supply the deficiency by prefixing decimal ciphers.

To multiply a decimal by 10, 100, 1000, etc.:—

Rule.—*Remove the decimal point as many places to the right as there are ciphers in the multiplier.*

DIVISION OF DECIMALS.

1. How many times are 2 tenths contained in 8 tenths?

2. How many times 5 hundredths in 15 hundredths? 6 thousandths in 24 thousandths?

3. Divide $\frac{9}{10}$ by $\frac{3}{10}$; .8 by .4; $\frac{24}{100}$ by $\frac{6}{100}$; .21 by .07

4. Divide .63 by .09; .063 by .007; .035 by .005

5. Of what order is the quotient when tenths are divided by tenths? Hundredths by hundredths?

6. Of what order is the quotient when any number is divided by a like number?

WRITTEN EXERCISES.

7. Divide 6.25 by .25; 62.5 by 2.5; .625 by .025

PROCESS.

(1) .25)6.25(25, *Ans.*
50
125
125

(2) 2.5)62.5(25, *Ans.*
50
125
125

(1) 25 hundredths are contained in 625 hundredths (a like number) 25 times. The quotient is an integer.

(2) 25 tenths are contained in 625 tenths 25 times. The quotient is an integer.

(3) 25 thousandths are contained in 625 thousandths 25 times. The quotient is an integer.

8. Divide 17.28 by .24; 1.728 by .048

9. Divide 94.08 by .84; 9.408 by .084

10. Divide .625 by .25; by 2.5

PROCESS.

(1) .25).625(2.5, *Ans.*
50
125
125

(2) 2.5).625(0.25, *Ans.*
50
125
125

(1) 25 hundredths are contained in 62 hundredths (a like number) 2 times, and hence the first quotient figure is units, and the second is tenths.

(2) 25 tenths are contained in 6 tenths (a like number) 0 times, and hence the first quotient figure is 0 units and the second is 2 tenths. The quotient is 25 hundredths.

The 0 may be omitted.

11. Divide 3.528 by 12.6; by .42; by .84

12. Divide .6162 by .79; by .079; by 2.37

13. Divide 62.5 by .025

PROCESS.

.025)62.500(2500, *Ans.*
50
125
125
00

By annexing two decimal ciphers to 62.5, the dividend and divisor are made like numbers, and their quotient is an integer.

14. Divide 25.6 by .032; by .16; by .064

15. Divide 2.5 by 1.25; by .0125; by .025

16. Divide 256.7 by 3.02; by .0302; by .604
17. Divide 80.5 by .35; by .00035; by .0115
18. Divide 402.5 by 1.75; by .0175.
19. Divide 80.025 by .275; by 2.75
20. Divide 40.25 by 17.5; by .175
21. Divide 51.2 by .64; by .064
22. Divide .4704 by 4.2; by .42
23. Divide 3.456 by .96; by 9.6
24. Divide .5184 by .48; by .048
25. Divide 306 by 40.8; by .408
26. 52.5 by 62½
27. .633 by 8.4⅖
28. .2832 by 4.4¼
29. .768 by 6.4
30. 76.8 by .096
31. 8.75 by .12½
32. 3.85 by 8.7½

33. Divide the dividend in each of the above problems, from 19 to 32, by .025; by 2.5

To Teachers.—Show that *the dividend contains as many decimal places as both divisor and quotient;* and that the accuracy of the quotient found may be tested by multiplying it by the divisor.

34. Divide 34.5 by 10; by 100; by 1000.

The removal of the decimal point one place to the left divides the number by 10; two places to the left by 100; three places to the left by 1000, etc.

PROCESS.
34.5 ÷ 10 = 3.45
34.5 ÷ 100 = .345
35.5 ÷ 1000 = .0345

35. Divide 436.7 by 100; by 1000; by 10000.
36. Divide 234.6 by 100; by 1000; by 100000.

To divide one decimal by another:—

Rule.—*Divide as in the division of integers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

Notes.—1. When the divisor contains more decimal places than the dividend, *annex* decimal ciphers to the dividend.

2. When the quotient has not enough decimal figures, *prefix* decimal ciphers.

To divide a decimal by 10, 100, 1000, etc.:—

Rule.—*Remove the decimal point as many places to the left as there are ciphers in the divisor.*

REVIEW PROBLEMS.

1. Express decimally 16½ thousandths.

2. Change .325 to a common fraction.

3. Change $\frac{8}{25}$ to a decimal.

4. Change 16¾ to a mixed decimal.

5. From 56.675+48.12½ take 5.0875

6. Multiply 37.564 by 4.05; by 25.5

7. Divide 84.564 by 9.72; by .0012

8. Divide .03672 by .036; by 360.

9. If a man travel 3.25 miles in an hour, how many miles will he travel in 12.4 hours?

10. If a steamer sail 12.5 miles per hour, in how many hours will it sail 112.5 miles?

11. If 3.5 acres produce 141.4 bushels of corn, what is the yield per acre?

12. The product of two numbers is 6.23, and one of the numbers is 124.6: what is the other number?

QUESTIONS FOR REVIEW.

What is meant by the decimal division of a unit? What is a decimal fraction? What is its denominator?

In how many ways may a decimal fraction be expressed in figures? Which is called the common form? Which is called the decimal form?

What is a mixed decimal? For what is the decimal point used?

What is the first decimal order? The third? The sixth?

Does the value of the decimal orders increase or decrease from left to right? From right to left?

How are decimals reduced to common fractions? Common fractions to decimals?

How are decimals added? How are they subtracted?

How is one decimal multiplied by another? How many decimal places are in the product?

How is one decimal divided by another? How many decimal places are in the quotient? What is done when the divisor contains more decimal places than the dividend?

UNITED STATES MONEY.

UNITED STATES money is the legal currency of the United States.

It consists of *coin* and *paper money.* Coin is called *specie currency*, or *specie*, and paper money *paper currency.*

The principal gold coins are the double eagle ($20), eagle ($10), half-eagle ($5), quarter-eagle ($2½), three-dollar piece, and dollar.

The silver coins, now being coined, are the dollar, half-dollar, quarter-dollar, and dime.

The five-cent piece, now coined, is made of nickel and copper. The cent and two-cent pieces are made of bronze, an alloy of copper, tin, and zinc.

Paper money consists of notes issued by the United States, called *treasury notes*, and bank-notes issued by National Banks.

TO TEACHERS.—If practicable, secure for illustration the coins and notes in common use.

The denominations of United States money are *dollars*, *dimes*, *cents*, and *mills*. There are 10 mills (*m.*), in 1 cent (*ct.*), 10 cents in 1 dime (*d.*), and 10 dimes in 1 dollar ($). There are 100 cents in a dollar.

The denominations used in accounts are *dollars* and *cents*. Mills are used only in making calculations.

ORAL EXERCISES.

1. How many mills in 1 cent? In 5 cents? 4 cents? 7 cents? 9 cents? 10 cents?

2. How many cents in 1 dime? In 4 dimes? 5 dimes? 8 dimes? 10 dimes? 15 dimes?

3. How many dimes in 1 dollar? In 3 dollars? 6 dollars? 4 dollars? 8 dollars? 10 dollars?

4. How many cents in 10 mills? In 50 mills? 40 mills? 60 mills? 80 mills? 100 mills?

5. How many dimes in 40 cents? In 90 cents? 60 cents? 80 cents? 70 cents? 100 cents?

6. How many dollars in 50 dimes? In 70 dimes? 60 dimes? 90 dimes? 80 dimes? 100 dimes?

United States money is written on the decimal scale. The first two figures at the right of dollars denote cents, and the third figure denotes mills. The figures denoting dollars are preceded by the *dollar sign* ($), and they are separated from those denoting cents by the decimal point, called a *separatrix*. Thus, $45.307 denotes 45 *dollars* 30 *cents* 7 *mills*.

7. Copy and read $3.45; $3.506; $1.055; $0.75

8. Copy and read $3.08; $9.005; $0.725; $0.075; $15.05

9. Read $100; $405.50; $750.15; $60.065; $0.625

Cents and mills are decimal parts of a dollar, cents being *hundredths* and mills *thousandths*. $45.307 is a mixed decimal, the dollars being expressed by integral orders, and the cents and mills by decimal orders. It may be read 45 *dollars and* 307 *thousandths of a dollar*, or, more briefly, 45 *and* 307 *thousandths dollars*.

10. Read the sums of money in examples 7, 8, and 9 as decimal numbers.

WRITTEN EXERCISES.

11. Write in figures 4 dollars 45 cents 5 mills.

12. Write in figures 60 dollars 6 cents 4 mills.

13. Write in figures 75 cents; 40 cents 8 mills.

14. Write in figures 300 dollars 3 cents 7 mills.

15. Write in figures 50 dollars 5 mills; 50 cents 6 mills.

16. How many cents in $15? *Ans.* 1500 cts.

17. How many cents in $75? $108? $60? $63? $500? $150? $100? $24? $70? $125? $108?

Solution.—In $1 there are 100 cents, and in $15 there are 15 times 100 cents, or 1500 cents.

Note.—Dollars are reduced to cents by annexing two ciphers, and to mills by annexing three ciphers.

18. How many cents in $12.65? *Ans.* 1265 cts.
19. How many cents in $6.40? $10.05? $20.04? $10.40?
20. How many mills in $25? *Ans.* 25000 mills.
21. How many mills in $2? $10? $50? $35?
22. How many mills in $4.375? *Ans.* 4375 mills.
23. How many mills in $3.125? $4.50? $6.12½? $4.37½?

NOTE.—Dollars and cents are reduced to cents, and dollars, cents, and mills to mills, by removing the separatrix and the dollar sign.

24. How many dollars in 7500 cents? *Ans.* $75.
25. How many dollars in 15000 mills? *Ans.* $15.
26. Reduce 375 cents to dollars; 1375 mills to dollars.
27. Reduce $4.50 to cents; $5.16 to mills.
28. Reduce $.37½ to mills; 1054 mills to dollars.
29. Reduce $.06¼ to mills; 506 cents to dollars.

ADDITION AND SUBTRACTION.

WRITTEN PROBLEMS.

1. What is the sum of $50, $16.50, $3.333, and $.87½?

Write the several numbers to be added so that units of the same denomination and order stand in the same column, and add as in simple numbers.

The dollar sign need be written but twice,—before the first additive number and before the answer.

PROCESS.
$50.
16.50
3.333
.875
$70.708, *Ans.*

2. What is the sum of $1.20, $5, $10.15, $.85, $.62½?

3. What is the sum of $9, $12.50, $4.37½, $40.08, $6.33, $8.75, $13.12½, $24.62½, and $25?

4. Add $45.37½, $100.50, $16.12½, $37, $9.05, $.87½, $4.44, $9.63, $43.62½, $1.08, $40.87½, $16.50, $37.62½, and $95.

5. From $37.50 take $5.62½.
6. From $6.37½ take $5.87¾.
7. From $100 take $1.256
8. From $10 take $.10

(5) PROCESS.
$37.500
5.625
$31.875, *Ans.*

9. A man sold a carriage for $160.75, a horse for $125, a set of harness for $26.37½, and a saddle for $15.62½: what was the amount received?

10. A grocer buys flour at $8.62½ a barrel, and sells it at $10 a barrel: what is his gain?

11. A merchant paid $32.50 for a barrel of sugar, and sold it for $35: how much did he gain?

12. A man earns $17.50 a week, and his expenses are $12.62½ a week: how much can he save each week?

13. A man bought a house and lot for $3506.75, and sold it for $4000: what was his gain?

14. A man bought a carriage for $160, paid $22.75 for repairing it, and then sold it for $180: how much did he lose?

15. Mr. Smith bought a house and lot for $4500, paid $40.50 for a fence, $105.65 for painting, $47.12 for papering, and $25 for other improvements: what was the cost of the property? What will Mr. Smith gain, if he sell it for $5600?

To add or subtract sums of money:—

RULE.—*Write units of the same denomination and order in the same column, add or subtract as in simple numbers, separate dollars and cents by the decimal point, and prefix the dollar sign.*

MULTIPLICATION AND DIVISION.

WRITTEN PROBLEMS.

1. What will 9 cords of wood cost at $3.62½ a cord?

(1) PROCESS.

$$\begin{array}{r} \$3.625 \\ 9 \\ \hline \$32.625,\ \textit{Ans.} \end{array}$$

2. What will 16 barrels of flour cost at $7.50 a barrel?

3. What will 40 yards of cloth cost at $1.12½ a yard? At $1.33⅓? At $1.40?

4. What will 12⅗ tons of hay cost at $11.50 a ton?

5. At 18¾ cents a dozen, what will 12 dozen eggs cost? 20 dozen? 36 dozen? 45 dozen?

6. If a boy earn $4.37½ a week, how much will he earn in 16 weeks? In 32 weeks? 40 weeks?

7. A drover sold 36 cows at $33.33⅓ a head: how much did he receive for them?

8. What will 90 bushels of wheat cost at $1.62½ a bushel? At $1.12½? At $1.18¾?

9. If 9 cords of wood cost $32.62½, what will 1 cord cost?

(9) PROCESS.
9)$32.625
$3.625, *Ans.*

10. If 18 pounds of sugar cost $2.16, what will 1 pound cost?

11. A man paid $1687.50 for 45 acres of land: what was the price per acre?

12. A grocer paid $135 for 18 barrels of flour: what was the cost a barrel?

13. A man earned $90 in 8 weeks: how much did he earn each week?

14. At $.12½ a dozen, how many dozens of eggs can be bought for $5?

Reduce both divisor and dividend to like numbers, *i. e.*, to *mills.*

$.12½ = $.125, or 125 mills;
$5 = $5.000, or 5000 mills.

PROCESS.

mills. mills.
125)5000(40
500 40 doz., *Ans.*
0

Or—
$.125)$5.000(40
5 00 40 doz., *Ans.*
0

15. At $1.25 a bushel, how many bushels of corn can be bought for $75?

16. At 3 cents apiece, how many lemons can be bought for $1.20? For $3? For $6?

17. If a boy earn 75 cents a day, in how many days will he earn $6.75? $13.50? $24? $75?

18. At 37½ cents a bushel, how many bushels of oats can be bought for $15? For $56.25? $91.50?

To multiply or divide sums of money:—

RULE.—*Multiply or divide as in simple numbers, separate dollars and cents in the result by the decimal point, and prefix the dollar sign.*

NOTE.—To divide one sum of money by another, both numbers must first be reduced to *like* numbers.

When the price of an article is a fractional part of a dollar, the cost of two or more articles may be readily found by taking the fractional part of the cost at $1.

The fractional parts of a dollar most commonly used in business, called *aliquot parts*, are:—

50 cts. = $\frac{1}{2}$ of \$1.00
25 cts. = $\frac{1}{4}$ of \$1.00
20 cts. = $\frac{1}{5}$ of \$1.00
10 cts. = $\frac{1}{10}$ of \$1.00

$12\frac{1}{2}$ cts. = $\frac{1}{8}$ of \$1.00
$6\frac{1}{4}$ cts. = $\frac{1}{16}$ of \$1.00
$33\frac{1}{3}$ cts. = $\frac{1}{3}$ of \$1.00
$16\frac{2}{3}$ cts. = $\frac{1}{6}$ of \$1.00

19. What will 56 lb. of grapes cost at $12\frac{1}{2}$ cts. a pound?

WRITTEN PROCESS.
8)\$56, at \$1.
\$7, at $12\frac{1}{2}$ cts.

SOLUTION.—At \$1 a pound, 56 pounds will cost \$56, and, at $12\frac{1}{2}$ cts. a pound, they will cost $\frac{1}{8}$ of \$56, which is \$7.

20. What will 120 readers cost at $33\frac{1}{3}$ cts. apiece? At 25 cts.? At 20 cts.? At $16\frac{2}{3}$ cts.?

21. What will 24 dozens of eggs cost at $16\frac{2}{3}$ cts. a dozen? At $12\frac{1}{2}$ cts.? At 20 cts.? At 25 cts.?

MISCELLANEOUS PROBLEMS.

1. A grocer paid \$6 for a barrel of flour and sold it for \$7.62: how much did he gain?

2. A man bought apples at \$3 a barrel and sold them at \$2.65 a barrel: how much did he lose?

3. A fruit-dealer bought 22 quarts of berries for \$1.50 and sold them at 10 cts. a quart: how much did he gain?

4. What will 24 pounds of sugar cost at 12½ cts. a pound? At 16⅔ cts.? At 20 cts.?

5. What will 15 pounds of coffee cost at 20 cts. a pound? At 25 cts.? At 33⅓ cts.?

6. How many pounds of butter at 16⅔ cts. a pound can be bought for 50 cts.? For $2? For $9?

7. A fruit-dealer makes a profit of 20 cts. on each bushel of apples which he sells: how many bushels must he sell to make $5? To make $24?

8. A boy pays $2.50 for 100 papers, and sells them at 5 cents apiece: how much does he make?

9. At $2.50 a day, how many days must a man work to earn $25? To earn $120?

10. How many yards of cloth, at 12½ cts. a yard, can be bought for a five-dollar bill?

WRITTEN PROBLEMS.

11. A man's State tax was $27.60, city tax $68.25, and other taxes $7.58: what was the amount of his taxes?

12. A man bought a house and lot for $5400, and, after expending $1500 for improvements, sold the property for $7500: how much did he gain?

13. What will 8.75 tons of coal cost at $4.50 a ton? At $5.12½ a ton? At $4.62½ a ton?

14. What will be the cost of digging a cellar of 437.24 cubic yards at $.65 a cubic yard?

15. What is the cost of 35 reams of paper, weighing 44 pounds each, at 18 cents a pound?

16. How many yards of carpeting, at $1.75 a yard, can be bought for $87.50? For $350?

17. At 16⅔ cts. a pound, how many pounds of cheese can be bought for $1? For $3? For $2.50?

18. A fruit-dealer sold 144 baskets of peaches for $252: what was the price per basket?

19. If 8 barrels of salt cost $36, what will 1 barrel cost? 13 barrels? 52 barrels? 64 barrels?

20. A farmer sold 35 pounds of butter at 20 cents a pound, and with the money bought muslin at 12½ cents a yard: how many yards of muslin did he buy?

21. A grain-dealer bought 15000 bushels of wheat at $1.35 a bushel, and sold it the next week for $1.48 a bushel: what was his gain?

22. A drover bought 240 sheep at $4.50 a head, drove them to market at an expense of $75, and then sold them at $6.50 a head: how much did he gain?

23. A book-seller sold 12 geographies, at $1.25; 20 readers, at $.85; 30 arithmetics, at $.65; and 45 spellers, at $.30: what was the amount of the bill?

24. A farmer sold 200 bushels of wheat, at $.95 a bushel; 500 bushels of corn, at 58 cents a bushel; 65 bushels of potatoes, at 60 cents a bushel; 12 tons of hay, at $12.50 a ton; and 225 pounds of butter, at 20 cents a pound: what was the amount of his product?

25. A man bought 250 bushels of coal, at 15 cents a bushel; 7 cords of wood, at $5.50 a cord; 18 bushels of potatoes, at $.90 a bushel; and 9 barrels of apples, at $2.75 a barrel: how much did he pay for all?

BILLS.

A Bill of Goods is a written statement of goods sold. It gives the amount and price of each article, and the entire cost, and also the date and place of the sale, and the names of buyer and seller.

A bill is drawn against the buyer, or *debtor*, and in favor of the seller, or *creditor*.

A bill is receipted by writing the words "*Received payment*" at the bottom, and affixing the seller's name.

When sales are made at different times, the dates may be written at the left, as in bill 2.

1. CINCINNATI, O., JAN. 10, 1890.

MR. CHARLES WILSON,

Bought of JAMES COOPER & CO.

13 lbs.	Coffee,	@ 30c.	$3.90
4 lbs.	Butter,	@ 35c.	1.40
10 lbs.	Flour,	@ 6c.	.60
12 lbs.	Dried beef,	@ 24c.	2.88
25 lbs.	Sugar,	@ 18c.	4.50
3 lbs.	Starch,	@ 20c.	.60
	Received payment,		$.....

JAMES COOPER & CO.

What is the amount of the above bill?

Copy, fill, foot, and receipt each of the following bills:—

2. CHICAGO, ILL., JAN. 3, 1890.

THOS. M. COCHRANE,

Bought of PETER & BROTHERS.

1890.				
June 20,	27	yds. Brussels carpeting,	@ $2.60	
" "	23	yds. Ingrain "	@ 1.75	
" "	8¾	yds. Oil-cloth,	@ 1.20	
" "	32	yds. Curtains,	@ .60	
" 25,	27	yds. Matting,	@ 1.12½	
" "	16	Stair rods,	@ .22	
				$.....

Cr.

Sept. 15,	By 16 yds. Curtains returned.	$9.60	
" 18,	By Cash.	37.50	$.....
			$.....

Received payment,

3. NASHVILLE, TENN., OCT. 8, 1890.

SAMUEL MILLS,

To JONES, SMITH, & CO., *Dr.*

To	7	yds. Broadcloth,	@ $6.50	. . . . . .	
"	7¾	yds. Linen,	@ .90	. . . . . .	
"	2½	doz. Handkerchiefs,	@ 1.50	. . . . . .	
"	12½	yds. Muslin,	@ .18	. . . . . .	
"	9	yds. Muslin, bleached,	@ .33⅓	. . . . . .	
		Received payment,		$	

4. Andrew Wilson, bought of Smith & Waring, Cincinnati, O., Dec. 15, 1888, 7 gross shirt buttons, @ $4.50; 10 doz. linen napkins, @ $2.75; 8 doz. pairs kid gloves, @ $12.50; 3½ doz. linen handkerchiefs, @ $4.50; and 4 doz. pairs of socks, @ $2.25. Make out and receipt this bill, using bill 1 as a model.

5. John Adams, bought of Andrew Jackson, Cleveland, O., Feb. 10, 1890, 46 pounds of bacon, @ 15 cts.; 12½ pounds of dried beef, at 18 cts.; 62¼ pounds of lard, @ 12 cts.; 8 pounds of roast, at 11 cts.; and 13 pounds of steak, at 12½ cts. Make out and receipt this bill.

QUESTIONS FOR REVIEW.

What are the denominations of United States money in common use? In what denominations are accounts kept? For what are mills used? Where is the dollar sign written? How are dollars and cents separated? Is the separatrix a period or a comma?

What decimal part of a dollar are cents? Mills?

How are sums of money written to be added or subtracted? How are they added or subtracted? How is the result written?

How is a sum of money multiplied or divided by an abstract number? How is one sum of money divided by another?

Give the several aliquot parts of a dollar. How is the cost of two or more articles found by aliquot parts?

What is a bill of goods? What does it contain? How is a bill receipted? Where are the dates of sales written?

DENOMINATE NUMBERS.

DRY MEASURE.

Dry Measure is used in measuring grain, seed, fruit, vegetables, and many other dry articles.

The denominations are *pints*, *quarts*, *pecks*, and *bushels*.

TABLE.

2 pints (*pt.*) . . . are 1 quart . . . *qt.*
8 quarts . . . . . are 1 peck . . . *pk.*
4 pecks . . . . . are 1 bushel . . . *bu.*
1 bu. = 4 pk. = 32 qt. = 64 pt.

ORAL AND WRITTEN EXERCISES.

N. B.—The problems under most of the tables in Denominate Numbers are to be solved first orally and then on slate or paper.

1. How many pints in 3 quarts?

SOLUTION.—In 3 quarts are 3 times 2 pints, which is 6 pints.

2. How many pints in 5 quarts? In 15 quarts?
3. How many quarts in 10 pints? In 16 pints?

SOLUTION.—In 10 pints are as many quarts as 2 pints are contained times in 10 pints, which is 5. There are 5 quarts in 10 pints.

4. How many quarts in 3 pecks? 5 pecks? 6¼ pecks?
5. How many pecks in 16 quarts? 56 quarts?
6. How many pecks in 5 bushels? 7½ bushels?
7. How many bushels in 12 pecks? 32 pecks?
8. How many quarts in 8 pecks? In 24 pints?
9. How many pecks in 24 quarts? In 15 bushels?
10. How many pecks in 17 quarts? In 27 quarts?
11. How many bushels in 13 pecks? In 23 pecks?
12. How many pecks in 5 bushels? How many quarts? How many pints?

13. How many quarts in 10 bushels?

14. How many pints in 3 pecks?

15. How many bushels in 64 quarts?

16. What part of a quart is 1 pint? 3 pints?

17. What part of a peck is 1 quart? 3 quarts?

18. What part of a bushel is 1 peck? 2 pecks?

19. A man sold 1 bu. 3 pk. of clover-seed at 8 cents a quart: how much did he receive?

20. A fruit-dealer paid $7 for 3 bu. 3 pk. of peaches, and sold them at 60 cents a peck: what was his gain?

21. How many pecks of chestnuts can be bought for $15.60, at 40 cents a peck? How many bushels?

LIQUID MEASURE.

Liquid Measure is used in measuring liquids; as, oil, milk, alcohol, etc.

The denominations are *gills*, *pints*, *quarts*, and *gallons*.

TABLE.

4 gills (*gi.*)	are 1 pint	*pt.*
2 pints	are 1 quart	*qt.*
4 quarts	are 1 gallon	*gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

ORAL AND WRITTEN EXERCISES.

1. How many gills in 3 pints? In 15 pints?

2. How many pints in 5 quarts? 12½ quarts?

3. How many quarts in 5 gallons? 12½ gallons?

4. How many pints in 16 gills? 32 gills? 48 gills?

5. How many quarts in 12 pints? 48 pints?

6. How many gallons in 20 quarts? 32 quarts?

7. How many quarts in 15 pints? 21 pints?

8. How many pints in 8 quarts? 15½ quarts?

9. How many gallons in 8 quarts? 21 quarts?

10. How many quarts in 6 gallons? 12½ gallons?

11. What part of a gallon is 1 quart? 3 quarts?

12. How many quarts in ¾ of a gallon? ½ gallon?

13. How many pints in 5½ gallons? 10¼ gallons?

14. Reduce 15 gallons to quarts; to pints.

15. Reduce 36 pints to quarts; 72 pints to gallons.

16. Reduce 56 gallons to pints; 136 pints to gallons.

17. What will 3 gal. 5 qt. of oil cost at 8 cents a quart?

18. What will 10 quarts of milk cost at 5½ cents a pint?

19. If a gallon of wine cost $6, what will 1 pint cost?

20. If maple syrup cost $1.60 a gallon, what is the price per quart?

21. At 6 cents a quart, what will 5 gallons of milk cost? 4½ gallons? 10 gallons?

22. How many half-pint tumblers will 2 gallons of jelly fill? How many will 7 gallons fill?

23. How many vials, holding 2 gills each, can be filled from 5¼ gallons of alcohol?

24. A grocer bought 10 gallons of vinegar at 40 cents a gallon, and sold it at 15 cents a quart: how much did he gain?

25. A man bought 5 gallons of maple syrup at 80 cents a gallon, and sold it at 25 cents a quart: how much did he gain?

LONG MEASURE.

Long Measure is used in measuring lines or distances. It is also called *Linear Measure*.

The denominations in common use are *inches*, *feet*, *yards*, *rods*, and *miles*.

TABLE.

12 inches (in.) . . . are 1 foot . . . *ft.*
3 feet . . . . are 1 yard . . . *yd.*
5½ yards or 16½ feet . . are 1 rod . . . . *rd.*
320 rods . . . . are 1 mile . . . *mi.*
1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

NOTES.—1. The mile is commonly divided into halves, fourths, eighths, etc. The term *furlong* (⅛ of a mile) is now seldom used.

2. In measuring cloth, ribbons, etc., the width is not considered, and the yard is divided into halves, fourths, eighths, etc.

3. The *hand* is 4 inches, used in measuring the height of horses.

4. The *fathom* is 6 feet, used in measuring the depth of water.

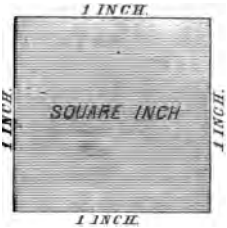
ORAL AND WRITTEN EXERCISES.

1. How many inches in 3 feet? 5 feet? 2½ ft.?
2. How many feet in 36 inches? 72 inches? 144 in.?
3. How many feet in 4 yards? 9 yd.? 12⅓ yd.?
4. How many yards in 15 feet? 21 ft.? 180 ft.?
5. How many yards in 2 rods? 6 rd.? 10 rd.?
6. How many feet in 2 rods? 6 rd.? 10 rd.?
7. How many rods in 2 miles? 5 mi.? ½ mi.?
8. What part of a foot is 4 inches? 6 in.? 9 in.?
9. How many inches in ¼ of a foot? ⅔ ft.? ¾ ft.?
10. What part of a mile is 80 rods? 160 rods?
11. How many rods in ⅛ of a mile? ⅝ mi.? ¾ mi.?
12. What part of a yard is 2 feet? 1½ ft.? 5 ft.?
13. A horse is 15 hands high: what is its height in feet?
14. A vessel sank in water 9 fathoms deep: what was the depth of the water in feet?

SQUARE MEASURE.

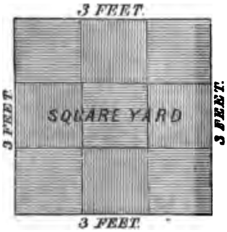
Square Measure is used in measuring surfaces. It is also called *Land Measure.*

The denominations are *square inches*, *square feet*, *square yards*, *square rods* (or *perches*), *acres*, and *square miles*.



A Square Inch is a square, each side of which is an inch in length.

NOTE.—The figure at the right represents a square inch of real size.



A Square Yard is a square, each side of which is a yard, or three feet, in length. It contains 9 square feet.

NOTE.—A square yard of real size may be drawn on the blackboard or floor and divided into nine square feet. The right angle, the rectangle, and the square should be taught objectively.

TABLE.

144	square inches (*sq. in.*)	are 1	square foot	*sq. ft.*
9	square feet	are 1	square yard	*sq. yd.*
30¼	square yards	are 1	square rod (or perch)	*sq. rd.*
160	square rods	are 1	acre	*A.*
640	acres	are 1	square mile	*sq. mi.*

ORAL AND WRITTEN EXERCISES.

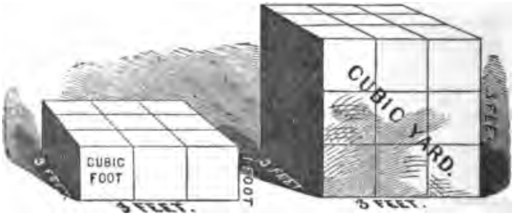
1. How many square feet in 5 sq. yd.? 9 sq. yd.?
2. How many square yards in 45 sq. ft.? 81 sq. ft.?
3. How many square inches in ¼ sq. ft.? $\frac{1}{12}$ sq. ft.?
4. How many square feet in ⅓ sq. yd.? ½ sq. yd.?
5. How many square rods in ½ of an acre? ¾ of an acre?

6. How many square feet in 3⅔ sq. yd.? 5⅓ sq. yd.?
7. How many acres in ½ of a square mile? ¼ sq. mi.?
8. How many square rods in 2 acres? 10 acres?
9. How many acres in 320 sq. rd.? 480 sq. rd.?
10. How many square rods in ½ acre? ¼ acre? ⅜ acre?
11. What part of an acre is 40 sq. rods? 120 sq. rd.?

CUBIC MEASURE.

Cubic Measure is used in measuring solids. It is also called *Solid Measure.*

The denominations are *cubic inches*, *cubic feet*, *cubic yards*, and *cords.*



To **Teachers.**—Teach the cube objectively, and show that the edges of a cubic inch, a cubic foot, and a cubic yard are respectively each one inch, one foot, and one yard.

TABLE.

1728 cubic inches (*cu. in.*). . are 1 cubic foot . . . *cu. ft.*
27 cubic feet . . . . . . are 1 cubic yard. . . *cu. yd.*
128 cubic feet . . . . . . are 1 cord . . . . . *cd.*

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

WRITTEN EXERCISES.

1. How many cubic inches in 10 cubic feet?
2. How many cubic inches in 24½ cubic feet?
3. How many cubic feet in 85 cubic yards?
4. How many cubic feet in 96⅓ cubic yards?

5. How many cubic feet in 15552 cubic inches?
6. How many cubic yards in 351 cubic feet?
7. How many cubic feet in 25⅛ cubic yards?
8. How many cords in 1536 cubic feet?

TIME MEASURE.

Time Measure is used in measuring time.

The denominations are *seconds*, *minutes*, *hours*, *days*, *years*, and *centuries*.

TABLE.

60 seconds (*sec.*) are 1 minute . . . . *min.*
60 minutes . . are 1 hour . . . . . *h.*
24 hours . . . are 1 day . . . . . *d* or *da.*
365¼ days. . . . are 1 solar year . . . *s. yr.*
100 years (365¼ d.) are 1 century . . . . *C.*

1 d. = 24 h. = 1440 min. = 86400 sec.

NOTES.—1. The common year contains 365 days, and since this is ¼ of a day (nearly) less than a solar, or true, year, one day is added to each fourth year, which is called *leap-year*. Leap-year contains 366 days, the additional day being given to February; and hence, in leap-years, February contains 29 days. The leap-years are exactly divisible by 4; as, 1884, 1888, 1892, etc. The exact length of a solar year is 365 d. 5 h. 48 min. 47⅓ sec.

2. The year is divided into twelve months, called calendar months. The names and order of the calendar months and the number of days in each are as follows: —

January,	1st	mo.,	31 days.	July,	7th	mo.,	31 days.
February,	2d	"	28(l. yr. 29 d.)	August,	8th	"	31 "
March,	3d	"	31 days.	September,	9th	"	30 "
April,	4th	"	30 "	October,	10th	"	31 "
May,	5th	"	31 "	November,	11th	"	30 "
June,	6th	"	30 "	December,	12th	"	31 "

The following couplet will assist in remembering the months which have 30 days each: —

> "Thirty days hath September,
> April, June, and November."

ORAL EXERCISES.

1. How many hours in 3 days? In $\frac{3}{4}$ of a day?

2. How many minutes in 2 hours? $\frac{1}{2}$ h.? $2\frac{1}{2}$ h.?

3. How many seconds in 5 minutes? In $\frac{2}{3}$ min.?

4. How many days in 72 hours? 96 h.? 240 h.?

5. How many hours in 120 minutes? 300 min.?

6. There are 7 days in a week: how many days in 4 weeks? 12 weeks? $5\frac{3}{7}$ weeks?

7. What part of a day is 6 hours? 12 h.? 18 h.?

8. What part of an hour is 30 minutes? 45 min.?

9. How many seconds in $\frac{1}{4}$ of a minute? In $\frac{1}{2}$ min.?

10. What part of a minute is 20 seconds? 40 sec.?

WRITTEN EXERCISES.

11. How many minutes in $15\frac{1}{2}$ hours?

12. How many hours in 2880 minutes?

13. How many hours in a common year?

14. How many hours in a leap-year? How many minutes? How many seconds?

15. How many hours in a solar year? How many minutes? How many seconds?

16. How many hours will there be in the month of February, 1890? In February, 1892?

17. If your pulse beat 75 times a minute, how many times will it beat in 1 day? In $365\frac{1}{4}$ days?

18. How many days in the first six months of a common year? In the last six months?

19. How many hours in the month of June? In October? In November?

20. How many hours in the three spring months, March, April, and May?

21. A man is 80 years of age: how many days has he lived, allowing $365\frac{1}{4}$ days to the year?

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is used in weighing all articles except gold, silver, and precious stones.

The denominations are *ounces*, *pounds*, *hundred-weights*, and *tons*.

TABLE.

16 ounces (*oz.*) . . . are 1 pound. . . . . . *lb.*
100 pounds. . . . . are 1 hundred-weight . . *cwt.*
20 hundred-weight . are 1 ton. . . . . *T.*
1 T. = 20 cwt. = 2000 lb. = 32000 oz.

ORAL AND WRITTEN EXERCISES.

1. How many ounces in 2 pounds? 6 lb.? 1$\frac{3}{4}$ lb.?

2. How many pounds in 32 ounces? 48 oz.? 96 oz.?

3. How many ounces in 2 pounds? 5$\frac{1}{2}$ lb.? 10$\frac{3}{4}$ lb.?

4. How many cwt. in 3 tons? 8 tons? 5$\frac{3}{4}$ tons?

5. How many ounces in $\frac{3}{4}$ of a pound? $\frac{5}{8}$ lb.? $\frac{7}{8}$ lb.?

6. How many cwt. in $\frac{1}{4}$ of a ton? $\frac{3}{4}$ T.? $\frac{3}{5}$ T.?

7. A barrel of flour contains 196 pounds: how many pounds in $\frac{1}{2}$ of a barrel? $\frac{3}{4}$ of a barrel?

8. A barrel of pork contains 200 pounds: how many barrels will contain a ton of pork?

To Teachers.—Since Troy Weight is used chiefly by jewelers, and Apothecaries Weight by physicians and druggists, their study may well be deferred until they are reached in the *New Complete Arithmetic.* The tables are, however, here given for use, if this be thought best. The teacher can readily dictate both oral and written exercises.

TROY WEIGHT.

24 grains (*gr.*)	**are 1 pennyweight**	***pwt.***
20 pennyweights	**are 1 ounce**	***oz.***
12 ounces	**are 1 pound**	***lb.***

APOTHECARIES WEIGHT.

20 grains (*gr.*)	**are 1 scruple**	**℈**
3 scruples	**are 1 dram**	**ʒ**
8 drams	**are 1 ounce**	**℥**
12 ounces	**are 1 pound**	***lb.***

MISCELLANEOUS TABLE.

The following denominations are in common use:—

Paper.	*Numbers.*
24 sheets are 1 quire.	**12 things are 1 dozen.**
20 quires are 1 ream.	**12 dozen are 1 gross.**
2 reams are 1 bundle.	**20 things are 1 score.**

ORAL AND WRITTEN EXERCISES.

1. How many sheets of paper in 2 quires? 5½ quires?

2. How many quires of paper in 4 reams? 6⅖ reams?

3. How many eggs in 5 dozen? 7¾ dozen? 8⅓ dozen?

4. How many years are 3 score years? 3 score years and 10? 4 score and 10 years?

5. How many crayons are there in 10 boxes, if each box contains 1 gross? In 25 boxes?

6. If a vest require 6 buttons, how many vests will 1 gross of buttons trim? 5 gross? 20 gross?

MISCELLANEOUS WRITTEN PROBLEMS.

1. How many yards in $16\frac{3}{5}$ miles? In $37\frac{1}{2}$ miles?

2. How many acres in a township 6 miles square?

3. A stack of hay contains 9000 pounds: what will it cost at $12 a ton? At $17.50 a ton?

4. What will 40 miles of telegraph wire cost at 75 cents a rod? At $87\frac{1}{2}$ cents a rod?

5. How many gross of pens will supply 4320 pupils one year, if each pupil require 4 pens?

6. How many steps of 2 ft. 6 in. each will a man take in walking 2 miles? In walking $\frac{3}{4}$ of a mile?

7. How many times will a wheel 6 feet in circumference turn round in going $2\frac{1}{2}$ miles? 4 miles?

8. Sound travels 1090 feet a second: how many miles will it travel in 60 seconds?

9. How many rods of fence will be required to inclose a farm which is $\frac{1}{4}$ of a mile long and $\frac{1}{8}$ of a mile wide?

10. How many times will a clock that ticks seconds, tick in the month of June? In the month of July?

11. If a person read a half-hour each day, how many hours will he read in 40 years, of $365\frac{1}{4}$ days each?

12. How many jugs, each holding 1 gal. 2 qt., can be filled from a barrel of vinegar containing $31\frac{1}{2}$ gallons?

13. A boy bought 5 pecks of cherries at 60 cents a peck, and sold them at 10 cents a quart: how much did he gain?

14. A fruit-dealer bought 12 bushels of strawberries at $2.25 a bushel, and sold them at 10 cents a quart: how much did he gain?

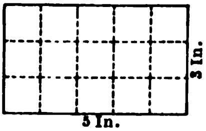
15. A grocer bought 6 barrels of vinegar, containing $31\frac{1}{2}$ gallons each, at $6.50 a barrel, and sold it at 10 cents a quart: how much did he gain?

16. If a milk-dealer sell daily 7 cans of milk, each holding 12 gal. 2 qt., how many gallons of milk does he sell in 28 days? In 40 days?

MENSURATION.

MEASUREMENT OF SURFACES.

1. How many square inches in a piece of paper 5 inches long and 3 inches wide?



Solution.—In a piece 5 inches long and 1 inch wide there are 5 square inches; and in a piece 3 inches wide there are 3 times 5 square inches, which is 15 square inches.

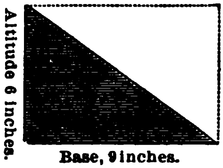
2. How many square inches in a piece of tin 10 inches long and 7 inches wide? 9 inches wide?

3. How many square feet in a piece of zinc 9 feet long and 5 feet wide? 10 feet long and $6\frac{1}{2}$ feet wide?

4. How many square feet in a board 16 feet long and $1\frac{1}{4}$ feet wide? 12 ft. long and $\frac{3}{4}$ ft. wide?

5. How many square inches in a triangular piece of tin whose base is 9 inches and altitude (height) 6 inches?

Solution.—Since the area of a triangle is one half the area of a rectangle with the same base and altitude, the piece of tin contains $\frac{1}{2}$ of 6 times 9 square inches, or $\frac{1}{2}$ of 54 square inches, which is 27 square inches.



6. How many square feet in a triangular flower bed whose base is 20 feet and altitude 10 feet?

WRITTEN PROBLEMS.

7. How many square yards in a garden 66 yards long and 60 yards wide?

8. How many square feet in a ceiling 45 feet long and $33\frac{1}{3}$ feet wide? How many square yards?

9. How many square rods in a field 56 rods long and 40 rods wide? How many acres?

(7) PROCESS.

$$\begin{array}{r} 66 \text{ sq. yd.} \\ \underline{60} \\ 3960 \text{ sq. yd.} \end{array}$$

10. How many square yards in a ceiling 16 yards long and 12 yards wide? 12 yd. long and $7\frac{2}{3}$ yd. wide?

11. How many acres in a field 60 rods long and 32 rods wide? 48 rods wide? 60 rods wide?

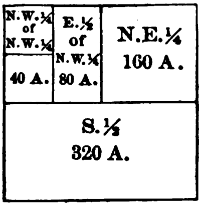
12. How many square feet in a floor 18 feet long and 15 feet wide? How many square yards?

13. How many square rods in a triangular garden whose base is $16\frac{1}{2}$ rods and altitude 12 rods?

14. How many acres in a triangular field whose base is $45\frac{3}{4}$ rods and altitude 32 rods?

15. How many acres in a street $2\frac{1}{2}$ miles long and 4 rods wide? 10 rods wide?

16. How many acres in a half-section of land? In a quarter-section?



NOTE.—Townships laid out by the United States government, since 1800, contain 36 square miles (6 miles square), and each square mile is called a **section.** Each section is divided into *half-sections, quarter-sections, half-quarter-sections*, etc. The diagram shows how a section is divided and the parts described. A section contains 640 acres.

17. How many acres of land in 3 half-sections? A quarter-section and a half-quarter-section together?

18. How many square yards in the four walls of a room 18 ft. long, $15\frac{1}{2}$ ft. wide, and 10 ft. high?

NOTE.—The surface of the walls of a room may be found *by adding the lengths of the four sides and multiplying the sum, changed to square units, by the height.*

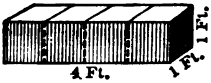
To find the area of a rectangle:—

RULE.—*Multiply the length, changed to square units, by the width.*

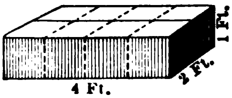
To find the area of a triangle:—

RULE.—*Multiply the base, changed to square units, by one half of the altitude.*

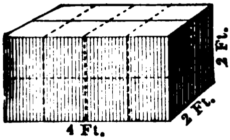
MEASUREMENT OF SOLIDS.



1. How many cubic feet in a block of stone 4 ft. long, 1 ft. wide, and 1 ft. thick?



2. How many cubic feet in a block 4 ft. long, 2 ft. wide, and 1 ft. thick?



3. How many cubic feet in a block 4 ft. long, 2 ft. wide, and 2 ft. thick?

Solution.—A block 4 ft. long, 1 ft. wide, and 1 ft. thick contains 4 cu. ft.; a block 4 ft. long, 2 ft. wide, and 1 ft. thick contains 2 times 4 cu. ft., which is 8 cu. ft.; and a block 4 ft. long, 2 ft. wide, and 2 ft. thick contains 2 times 8 cu. ft., which is 16 cu. ft.

4. How many cubic feet in a block 4 ft. long, 3 ft. wide, and 2 ft. thick?

5. How many cubic feet in a cubic block 3 ft. long, 3 ft. wide, and 3 ft. thick?

6. How many cubic inches in a brick 8 in. long, 4 in. wide, and 2 in. thick?

7. How many cubic inches in a block of silver 6 in. long, 2 in. wide, and 1½ in. thick?

8. How many cubic feet in a box 5 ft. long, 4 ft. wide, and 3 ft. deep? 4 ft. long, 2½ ft. wide, and 2 ft. high?

9. How many cubic feet in a pile of wood 10 ft. long, 4 ft. wide, and 5 ft. high?

10. How many cubic feet in a brick wall 50 ft. long, 4 ft. high, and 2 ft. thick?

11. How many cubic feet in a block of marble 10 ft. long, 1½ ft. wide, and 6 inches thick?

12. How many cubic feet in a room 20 ft. long, 12 ft. wide, and 10 ft. high?

WRITTEN PROBLEMS.

13. How many cubic ft. in a block of marble 15 ft. long, 12 ft. wide, and 7 ft. thick?

A block 15 ft. by 1 ft. by 1 ft. contains 15 cu. ft.; a block 15 ft. by 12 ft. by 1 ft. contains 12 times 15 cu. ft., or 180 cu. ft.; a block 15 ft. by 12 ft. by 7 ft. contains 7 times 180 cu. ft., which is 1260 cu. ft.

PROCESS.

$$\begin{array}{r l} 15 & \text{cu. ft.} \\ \underline{12} & \\ 180 & \text{cu. ft.} \\ \underline{7} & \\ 1260 & \text{cu. ft.} \end{array}$$

14. How many cubic feet in a rectangular rock 18 ft. long, 13 ft. wide, and 8 ft. high?

15. How many cubic feet in a pile of wood 24 ft. long, 6 ft. wide, and 8 ft. high?

16. How many cubic feet in a cellar 44 ft. long, 27 ft. wide, and 5 ft. deep? How many cubic yards?

17. How many cubic feet of water does a cistern hold which is 5 ft. square and 3½ feet deep?

18. How many cubic feet of air in a school-room 36 ft. long, 25 ft. wide, and 12 ft. high? If there are 54 persons in the room, how many cubic feet of air to each person?

19. How many cubic yards of earth must be removed to excavate a cellar 36 ft. long, 28½ ft. wide, and 6 ft. deep? What will be the cost at 33⅓ cts. a cubic yard?

20. How many cords of wood in a pile 48 ft. long, 4 ft. wide, and 4 feet high? 6 ft. high? 8 ft. high?

21. What will be the cost of a pile of four-foot wood 40 ft. long and 6 ft. high, at $4.50 a cord?

NOTE. — A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high contains 128 cu. ft. or 1 cord.

To find the solid contents of a rectangular solid: —

RULE.—*Multiply the length, changed to cubic units, by the width, and this product by the height.*

PERCENTAGE.

ORAL EXERCISES.

ONE hundredth of a number is one per cent of it; two hundredths, two per cent; three hundredths, three per cent; and so on.

NOTE.—The term per cent is from the Latin *per centum*, which means *by the hundred.*

1. What per cent of a number is $\frac{5}{100}$ of it? $\frac{8}{100}$ of it? $\frac{15}{100}$ of it? $\frac{33}{100}$ of it? $\frac{100}{100}$ of it?

2. What per cent of a number is .04 of it? .06? .08? $.07\frac{1}{3}$? $.00\frac{1}{3}$? $.00\frac{1}{2}$? $.12\frac{1}{4}$?

The character % is called the *per cent sign.* It is used for the words *per cent.* Thus 5% denotes 5 per cent, $3\frac{1}{3}$% denotes $3\frac{1}{3}$ per cent, $\frac{1}{3}$% denotes $\frac{1}{3}$ of one per cent.

3. How many hundredths of a number is 3% of it? 4% of it? 6%? 8%? 12%? 22%? 75%?

4. How many hundredths is $5\frac{1}{2}$%? $\frac{1}{2}$%? $7\frac{1}{3}$%? $\frac{3}{4}$%?

WRITTEN EXERCISES.

5. Write decimally 4%; $\frac{1}{3}$%; $6\frac{1}{2}$%; 125%.

PROCESS.—4% = .04; $\frac{1}{3}$% = $.00\frac{1}{3}$; $6\frac{1}{2}$% = $.06\frac{1}{2}$; 125% = 1.25

6. Write decimally 16%; 5%; $7\frac{1}{2}$%; $8\frac{1}{3}$%; $6\frac{1}{4}$%; $\frac{1}{2}$%; $\frac{3}{7}$%; $\frac{2}{3}$%; $33\frac{1}{3}$%; 75%; $133\frac{1}{3}$%; $\frac{1}{3}$%; 20%.

7. How much is 5% of 323? 8% of 254? $\frac{1}{3}$% of 385?

PROCESS:	(1) 5% = .05	(2) 8% = .08	$\frac{1}{3}$% = $.00\frac{1}{3}$
	323	254	585
	.05	.08	$.00\frac{1}{3}$
	16.15, *Ans.*	20.32, *Ans.*	1.95, *Ans.*

8. What is 8% of $3250? $5\frac{1}{3}$% of $390? $\frac{3}{5}$% of $525?

9. What is $3\frac{1}{3}$% of 321 miles? $\frac{1}{3}$% of 5280 feet?

10. What is 12½% of 2640 yards?

11. ½% of $450?
12. ⅔% of 365 days?
13. 8% of 6.45 feet?
14. 6⅓% of 96.6 miles?
15. 2⅕% of 120 acres?
16. 18¾% of 480 yards?
17. 33⅓% of $6.93?
18. 100% of $25.60?
19. 100% of any number?
20. ⅛% of 52.8 tons?

21. A man has an income of $2540, and his expenses are 62½% of his income: how much are his expenses?

22. A drover bought 245 sheep of A, and 60% as many sheep of B: how many sheep did he buy of B?

23. A man having 285 acres of land gave 33⅓% of it to his daughter: how many acres did she receive? How many acres had he left?

24. A school enrolled 160 pupils, and 55% of them were girls: how many girls were enrolled?

25. A school enrolled 320 pupils in a term, and the average number in daily attendance was 82½% of the number enrolled: what was the average number of pupils in daily attendance?

26. A school enrolled 775 pupils, and, on a certain day, 8% of the pupils were absent: how many pupils were absent? How many were present?

27. A farmer raised 2450 bushels of grain, and 42% of it was wheat, 24% oats, and the rest corn: how many bushels of each kind of grain did he raise?

28. A farmer raised in one season 650 bushels of grain, and 64% of it was corn: how many bushels of corn did he raise?

29. A drover bought 15 horses at $124 a head, and sold them at 30% profit: how much did he gain?

30. A merchant deducted for cash 5% from a bill of goods amounting to $540: how much did he receive for the goods?

31. A man asked $12500 for a farm, but sold it for 6% off for cash: what did he receive for the farm?

32. An agent sold a farm for $3500, and received a commission of 5%: how much did he receive?

NOTE.—The teacher should explain the business terms used in these problems, and add such information as will be of practical value to the pupil.

33. An attorney collected a debt of $324.50 and another debt of $75.50, and charged 10% for his services: what was his commission?

34. An architect furnished plans and superintended the erection of a building, which cost $25400, for 2½% of the cost: how much did he receive?

35. A commission merchant sold 320 barrels of flour at $6.50 a barrel, and charged 4½% commission: how much did he receive?

36. A broker bought $12300 worth of cotton, and charged ⅛%: what was his commission?

37. A store worth $25600 is insured for ¾ of its value, at ½%: what is the premium?

$$\text{PROCESS}: \begin{cases} \tfrac{3}{4} \text{ of } \$25600 = \$19200, \text{ amount insured.} \\ \$19200 \times .005 = \$96, \text{ premium.} \end{cases}$$

38. A house worth $7500 is insured for ⅔ of its value, at ⅜%: what is the premium?

39. A man has his life insured for $4000 at ⅞% per year: what is the annual premium?

40. A merchant imported a lot of silk, invoiced at $32600: what was the duty at 37½%?

41. A merchant imported 1500 yards of lace, invoiced at 62½ cents a yard: what was the duty at 40%?

42. What will be the cost of a draft on Chicago for $520 at ⅛% premium? At ⅛% discount?

43. For how much must teas that cost $.90, $1.00, and $1.10 a pound be sold to gain 20%? To gain 10%?

44. For how much must shoes that cost $2.40, $2.60, and $3.00 a pair be sold to gain 25%? To gain 20%?

INTEREST.

ORAL EXERCISES.

1. Mr. Jones borrowed $100 of a neighbor and paid him 6% of $100 for the use of the money for one year: how much did he pay?

The money paid for the use of money is called *interest.* The $6 which Mr. Jones paid for the use of $100 for one year was *the interest of* $100 *at* 6 *per cent.*

The money for the use of which interest is paid is called the *principal.* The $100 borrowed by Mr. Jones was the principal, and for the use of it he paid $6 interest.

2. What is the interest of $250 for one year at 4%? At 6%? At 5%? At 8%? At 10%? At 12%?

3. What is the interest of $120 at 5% for 1 year? 2 years? 3 years? 4½ years? 6⅓ years? 10 years?

Solution.—The interest of $120 at 5% for 1 year is $6, and the interest for 2 years is twice $6, which is $12; and so on.

4. What part of a year is 1 month? 2 months? 3 months? 4 months? 6 months? 8 months? 9 months?

WRITTEN EXERCISES.

5. What is the interest of $80 for 3 yr. 6 mo. at 6%?

6. What is the interest of $150 for 2 yr. 6 mo. at 5%?

7. $85.50 for 3 yr. at 6%?

8. $65 for 4 yr. 3 mo. at 8%?

9. $84 for 2 yr. 4 mo. at 7%?

10. $75.45 for 2 yr. at 6%?

11. $300.40 for 5 yr. at 10%?

12. $420 for 6 mo. at 5½%?

13. $96 for 1 yr. 6 mo. at 4%?

(5) PROCESS.

3 yr. 6 mo. = 3½ yr.

```
       $80
       .06
     $4.80, Int. for 1 yr.
        3½
     14.40, Int. for 3 yr.
      2.40, Int. for ½ yr.
    $26.80, Int. for 3½ yr.
```

14. $164 for 2 yr. 3 mo. at 8%? At 5%? At 6%?

15. $120 for 3 yr. 4 mo. at 5%? At $4\frac{1}{2}$%? At 10%?

To Teachers.—The above method is a convenient one for computing interest only for years and months.

THE SIX PER CENT METHOD.

1. What is the interest of $1 for 1 year at 6%? For 2 yr.? 3 yr.? 4 yr.? 5 yr.? 10 yr.?

Suggestion.— The interest of $1 for 1 yr. at 6% is $\frac{6}{100}$ of $1, which is 6 cts.; the interest for 2 yr. is 2 times 6 cts., which is 12 cts.; and so on.

2. What is the interest of $1 for 1 month at 6%? For 2 mo.? 4 mo.? 6 mo.? 8 mo.? 10 mo.?

Solution.—The interest of $1 at 6% for 1 month is $\frac{1}{12}$ of 6 cents, which is $\frac{1}{2}$ cent; for 2 months the interest is 2 times $\frac{1}{2}$ cent, which is 1 cent; for 4 months 4 times $\frac{1}{2}$ cent, or 2 cents; and so on.

The interest of $1 at 6% for any number of months is $\frac{1}{2}$ *as many cents as there are months.*

3. What is the interest of $1 for 3 months at 6%? For 5 mo.? 7 mo.? 9 mo.? 11 mo.?

4. What is the interest of $1 for 1 yr. 2 mo. at 6%? 1 yr. 4 mo.? 2 yr. 6 mo.? 2 yr. 8 mo.? 1 yr. 10 mo.?

5. What is the interest of $1 for 1 day at 6%? For 2 days? 3 days? 4 days? 6 days?

Solution.—Since the interest of $1 at 6% for 1 month, or 30 days, is $\frac{1}{2}$ cent or 5 mills, the interest for 1 day is $\frac{1}{30}$ of 5 mills, which is $\frac{1}{6}$ *of a mill.* The interest for 2 days is $\frac{2}{6}$ mill; 3 days $\frac{3}{6}$ mill; 4 days $\frac{4}{6}$ mill; 6 days $\frac{6}{6}$ mill, or 1 *mill.* The interest of $1 at 6% for any number of days is $\frac{1}{6}$ *as many mills as there are days.*

6. What is the interest of $1 for 12 days at 6%? 18 days? 24 days? 27 days? 19 days?

7. What is the interest of $1 for 13 days at 6%? 15 days? 21 days? 20 days? 14 days?

8. What is the interest of $1 at 6% for 1 mo. 6 da.? 2 mo. 12 da.? 4 mo. 15 da.? 2 mo. 20 da.?

9. What is the interest of $1 at 6% for 4 mo. 24 da.? 6 mo. 18 da.? 5 mo. 6 da.? 4 mo. 13 da.? 3 mo. 25 da.?

WRITTEN EXERCISES.

10. Find the interest of $1 at 6% for 2 yr. 3 mo. 14 da.

PROCESS.

Int. of $1 for 2 yr. = 6c. × 2 = 12c. = $.12
Int. of $1 for 3 mo. = ½c. × 3 = 1½c. = $.015
Int. of $1 for 14 da. = ⅙ m. × 14 = 2⅓ m. = $.002⅓
Int. of $1 for 2 yr. 3 mo. 14 da. = $.137⅓

The interest of $1 is found by taking *six times as many cents as there are years, one half as many cents as there are months*, and *one sixth as many mills as there are days.*

11. What is the interest of $1 at 6% for 2 mo. 24 da.?
12. For 1 yr. 9 mo. 6 da.?
13. For 2 yr. 1 mo. 25 da.?
14. For 3 yr. 3 mo. 20 da.?
15. For 1 yr. 1 mo. 24 da.?
16. 2 yr. 2 mo. 2 da.?
17. 1 yr. 1 mo. 1 da.?
18. 7 mo. 7 da.?
19. 5 yr. 5 da.?

20. Find the interest of $420 for 2 yr. 3 mo. 12 da., at 6%.

PROCESS.

Int. of $1 for —	$420
2 yr. = $.12	.137
3 mo. = $.015	2940
12 da. = $.002	1260
$.137	420
	$57.540, *Int.*

Since the interest of $1 at 6% for 2 yr. 3 mo. 12 da. is $.137, or .137 of $1, the interest of $420 for the same time is $.137 × 420, or $420 × .137, which is $57.54.

NOTE.—$.137 × $420 = $420 × .137, and hence the interest on any sum of money at $6% is found *by multiplying the principal by the interest of* $1 *for the time expressed as an abstract decimal.*

21. What is the interest of $145.60 for 1 yr. 5 mo. 24 da., at 6%? For 2 yr. 2 mo. 9 da.?

22. What is the interest at 6% of $64.20 for 1 yr. 3 mo.? For 5 mo. 9 da.?

23. $85.50 for 1 yr. 1 mo. 1 da.? For 9 mo. 27 da.?

24. $300.50 for 3 yr. 8 mo.? For 1 yr. 4 mo. 15 da.?

25. $500 for 1 yr. 1 mo. 18 da.? For 5 mo. 12 da.?

26. $450 for 2 yr. 2 mo. 2 da.? For 3 yr. 3 mo. 3 da.?

27. What is the interest of $31.20 from Oct. 23, 1885, to Apr. 12, 1889, at 8%?

PROCESS.

1889 yr. 4 mo. 12 da.
1885 " 10 " 23 "
3 yr. 5 mo. 19 da.

Int. of $1 at 6% for—
3 yr. = $.18
5 mo. = .025
19 da. = .003⅙
$.208⅙

$31.20
.208⅙
24960
6240
520
6)$6.49480, *Int. at 6%.*
$1.08246, *Int. at 1%.*
8
$8.65968, *Int. at 8%.*

28. What is the interest of $758.50 from Oct. 20, 1887, to May 14, 1890, at 6%? At 4%? At 10%?

29. What is the interest of $235.25 from May 13, 1889, to Apr. 1, 1890, at 6%? At 7%?

30. What is the amount of $240 from Feb. 15, 1886, to Apr. 27, 1889, at 6%? At 8%?

SUGGESTION.—The amount is the sum of principal and interest.

31. What is the amount of $137.50 from July 3, 1888, to Feb. 9, 1890, at 4%? At 5%?

32. A note of $243.75, dated June 8, 1883, was paid Nov. 14, 1885, with interest at 10%: what was the amount?

To compute interest at any rate per cent:—

RULE.—*Find the interest at 6 per cent, and then increase or diminish this interest by such a part of itself as will give the interest at the given rate.*

ANSWERS TO WRITTEN PROBLEMS.

N. B.—The answers to the problems in Part One are not given, and, in the succeeding Parts, when a problem has two or more answers, with few exceptions, *only the last answer is given.*

There is great advantage in having many problems in an elementary arithmetic without answers accessible to pupils; and it is not necessary for the teacher to be in possession of the answers, or, in their absence, to solve the problems.

The following plan of testing the accuracy of the answers obtained by pupils, has been successfully used by many teachers:—

When problems, the answers to which are not given, are recited in class, the teacher calls on a pupil to give his answer; and all the members of the class, who have the same answer, indicate this by raising the hand or otherwise. *If all agree*, the answer given may be accepted as correct, provided the problem has no ambiguity in statement or other special difficulty.

If a majority of the pupils agree, and the other pupils do not agree *with each other*, the answer obtained by the majority may usually be accepted as correct. When the pupils do not agree as to the answer, an analysis of the problem or a statement of the steps of the solution will disclose the error made.

PART TWO.—ADDITION.

Page 57.

15. 135693.
16. 240589.

Page 58.

17. 221566.
18. 270724.
19. 270527.
20. 307352.
21. 214353.
22. 435530.
23. 553002.
24. 57154930.
25. 24757307.
26. 30787140.
27. $3051.88.
28. $11446.28.
29. $79480.26.
30. $30204.09.

Page 59.

31. $5262.90.
32. $2212.59.
33. 4444844.
34. 389096.
35. 53293685.
36. 21716975.
37. 157973346.

Page 61.

24. 181 days.
25. 184 days.
26. 8996 pounds.

Page 62.

27. 14084 bushels.
28. 1949 voters.
29. 12326 youth.
30. 649 acres.
31. 265612 pounds.
32. $10585.
33. 1462 acres.
34. 1064 miles.
35. 1036 miles.

Page 63.

36. $2500.
37. 192975 sq. mi.
38. 66465 sq. mi.
39. $6515.

SUBTRACTION.

Page 66.

29. 8481.
30. 420135.
31. 39920612.
32. 48997976.

Page 67.

13. 354 pounds.
14. 685 bushels.
15. $2080.
16. 1992 bushels.
17. 478 youth.
18. 11597412.
19. $10339.

Page 68.

20. 156 years.
21. 325262 sq. mi.
22. 36795897 persons.
23. 128 years.
24. 59 years.
25. 6926 feet.
26. 5542 feet.
27. 91895 sq. mi.

Page 69.

6. $1171.
7. $1201.
8. $240.
9. 238720 sq. mi.
10. 318236 sq. mi.

MULTIPLICATION.

Page 71.

27. 27440670.
28. 13306938.
29. 29014132.
30. 165618574.
31. 142080798.
32. 13095430.
33. 1382950.

Page 72.

34. 5503716.
35. 2336565.
36. 2293760.
37. 34328125.
38. 40001688.
39. 92959677.
41. 2184288.
42. 4001448.
43. 18273600.
44. 13867680.
45. 23062642.
46. 30523415.
47. 284212341.
48. 216409884.
49. 89762000.
50. 175044000.
51. 153712800.
52. 138415250.
54. 288502000.
55. 15323600.
56. 252504000.
57. 260100000.
58. 625634000.
59. 498400000.
60. 404200000.
61. 108960000.
62. 214200000.
63. 355520000.
64. 12296000.
65. 25905600.
66. $8779200.
67. $2811200.

Page 75.

33. 8000 rods.
34. 68000 miles.
35. 33075 miles.
36. 9720 miles.
37. 2880 miles.
38. $14180.
39. 80640 min.
40. 330325 days.
41. 91520 pounds.
42. 2100 bushels.
43. $2295125.
44. 3194400 feet.

Page 76.

45. 8208000 miles.
46. 70000 times.
47. 68600 feet.
48. 115200 sheets.
49. 9360 pencils.
50. 40600 passengers.
51. 604800 seconds.
52. 16588800000 mi.
53. 94176000 feet.

54. 2304 miles.
55. \$112200.
56. \$225.
57. \$1080.
58. \$4440.
59. \$21120.

Page 77.

60. \$840.
61. \$1920.
62. 36180 men.
63. 149600 cents; or \$1496.00
64. \$9720.
65. \$80000.
66. \$3060.
67. \$900.
68. \$1330.
69. 476928 pounds.

DIVISION.

Page 81.

19. 13002.
20. 2202.
21. 25002.
22. 702.
23. 4804.
24. 241.
25. 97.
26. 594.
27. 15196.
28. 6125.
29. 58.
30. 81.
31. 4096.
32. 271.
33. 248.
35. $508\frac{15}{48}$.
36. $579\frac{360}{805}$.
37. $496\frac{3919}{5728}$.
38. $119\frac{693}{1806}$.
39. $209\frac{5620}{6785}$.
40. $768\frac{499}{2088}$.
41. 507.
42. $441\frac{144}{608}$.
43. 809.
44. 6007.

Page 82.

45. 750.
46. 144.
47. 54.
48. 75.
49. 1308.
50. 424.
51. 8.
52. 2460.
53. 3407.
54. 205.
55. 6666.
56. 3460.
57. 7054.
58. 305.
63. $384\frac{50}{1000}$.
64. $23\frac{45}{10000}$.
65. $450\frac{860}{1000}$.

Page 84.

21. 213 bushels.
22. 132 hours.
23. 486 days.
24. 81000 copies.
25. 243 barrels.
26. 1089 days.
27. 66 yards.
28. 4360 bushels.
29. 760 hogsheads.
30. 2430 boxes.

Page 85.

31. 47 farms.
32. 49 days.
33. 2470 years.
34. 36 hours.
35. 1512 weeks.
36. \$29154.
37. 207 days.
38. 603 days.
39. 640 barrels.
40. 2700 reams.

REVIEW.

Page 90.

26. 406.
27. 42909.
28. 22962.

Page 91.

29. 156.
30. 84.
31. 36.
32. 82532.
33. 1600.
34. \$14.
35. \$1870.
36. \$47.
37. \$2010.
38. \$625.
39. \$97260.
40. \$1128.
41. 107 hours.

Page 92.

42. \$64.
43. \$350.
44. \$55.

FRACTIONS.

Page 96.

17. $\frac{203}{4}$.
18. $\frac{187}{5}$.
19. $\frac{633}{10}$.
20. $\frac{462}{9}$.
21. $\frac{1463}{4}$.
22. $\frac{8266}{25}$.

Page 97.

15. $17\frac{3}{12}$, or $17\frac{1}{4}$.
17. $10\frac{10}{15}$, or $10\frac{2}{3}$.
18. $20\frac{12}{15}$, or $20\frac{4}{5}$.
19. $11\frac{1}{20}$.
20. $20\frac{2}{16}$, or $20\frac{1}{8}$.
21. 8.
22. $10\frac{12}{30}$, or $10\frac{2}{5}$.
25. $8\frac{5}{50}$, or $8\frac{1}{10}$.
26. $21\frac{2}{21}$.
27. $15\frac{10}{25}$, or $15\frac{2}{5}$.
28. $6\frac{4}{35}$.
30. $10\frac{5}{30}$, or $10\frac{1}{6}$.
33. $15\frac{25}{40}$, or $15\frac{5}{8}$.
34. 7.
35. $13\frac{25}{75}$, or $13\frac{1}{3}$.

Page 99.

18. $\frac{1}{2}$.
19. $\frac{2}{3}$.
20. $\frac{2}{3}$.
21. $\frac{3}{4}$.
22. $\frac{8}{9}$.
23. $\frac{7}{10}$.
24. $\frac{5}{7}$.
25. $\frac{3}{4}$.
26. $\frac{1}{5}$.
27. $\frac{3}{4}$.
28. $\frac{1}{3}$.
29. $\frac{3}{4}$.
30. $\frac{3}{11}$.
31. $\frac{1}{6}$.
32. $\frac{1}{4}$.
33. $\frac{1}{8}$.

Page 101.

20. $\frac{24}{30}$, $\frac{9}{30}$, $\frac{14}{30}$.
21. $\frac{9}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.
22. $\frac{12}{18}$, $\frac{15}{18}$, $\frac{10}{18}$.
23. $\frac{18}{30}$, $\frac{21}{30}$, $\frac{16}{30}$.
24. $\frac{12}{16}$, $\frac{14}{16}$, $\frac{9}{16}$.
25. $\frac{420}{84}$, $\frac{36}{84}$, $\frac{49}{84}$.
26. $\frac{4}{24}$, $\frac{10}{24}$, $\frac{7}{24}$.
27. $\frac{12}{30}$, $\frac{9}{30}$, $\frac{8}{30}$, $\frac{11}{30}$.
28. $\frac{16}{36}$, $\frac{30}{36}$, $\frac{22}{36}$, $\frac{15}{36}$.
29. $\frac{48}{60}$, $\frac{45}{60}$, $\frac{28}{60}$, $\frac{33}{60}$.

Page 102.

15. $1\frac{9}{10}$.
16. $2\frac{3}{8}$.
17. $1\frac{5}{24}$.
18. $1\frac{17}{30}$.
19. $1\frac{7}{10}$.
20. $1\frac{1}{15}$.
21. $1\frac{23}{24}$.
22. $2\frac{3}{10}$.
23. $1\frac{1}{4}$.
24. $\frac{3}{5}$.
25. $1\frac{5}{7}$.
26. $2\frac{1}{10}$.
27. $2\frac{7}{12}$.
28. $2\frac{16}{21}$.
29. $2\frac{11}{20}$.

Page 103.

31. $1039\frac{7}{12}$.
32. $175\frac{7}{8}$.
33. $181\frac{2}{3}$.
34. $344\frac{1}{10}$.
35. $823\frac{1}{4}$.
36. $3623\frac{11}{12}$.
37. $1164\frac{11}{21}$.
38. $3528\frac{1}{8}$.
39. $1180\frac{11}{24}$.

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14. $\frac{5}{24}$.
15. $\frac{13}{30}$.
16. $\frac{9}{50}$.
17. $\frac{11}{36}$.
18. $\frac{1}{42}$.
19. $\frac{1}{20}$.
20. $\frac{2}{45}$.
21. $\frac{1}{6}$.
22. $\frac{33}{50}$.
23. $\frac{7}{24}$.
24. $\frac{17}{42}$.
25. $\frac{11}{60}$.
27. $36\frac{5}{12}$.
28. $32\frac{3}{4}$.
29. $49\frac{2}{5}$.
30. $111\frac{1}{24}$.
31. $21\frac{1}{15}$.
32. $18\frac{1}{6}$.
33. $155\frac{7}{12}$.
34. $66\frac{2}{3}$.

1. $\frac{1}{12}$.
2. $\frac{1}{5}$.
3. $\frac{7}{12}$.

Page 105.

4. $7\frac{1}{8}$.
5. $\frac{7}{20}$ of money.
6. $\frac{4}{15}$ of farm.
7. $2\frac{1}{8}$ pounds.
8. $\frac{3}{8}$ acre.
9. $\$\frac{3}{8}$.
10. $\$\frac{1}{20}$.
11. $\frac{1}{4}$ of farm.
12. $1\frac{1}{4}$ tons.
13. $\frac{7}{24}$ bushel.
14. $\frac{4}{15}$ of pupils.

Page 107.

15. $48\frac{6}{7}$.
16. $69\frac{3}{8}$.
17. $118\frac{6}{11}$.
18. 260.
19. $206\frac{2}{3}$.
20. $222\frac{7}{8}$.
21. $346\frac{2}{3}$.
22. 252.
23. 176.
24. 175.
26. 900.
27. 1350.
28. 2220.
29. 3116.
30. $13533\frac{1}{3}$.
31. 23052.
32. 16848.
9. $3\frac{3}{20}$.
10. $7\frac{1}{2}$.

Page 108.

11. $59\frac{1}{2}$.
12. 100.

13. 1612.
14. 1036.
15. 5450.

Page 109.

16. $\frac{1}{4}$.
17. $\frac{1}{2}$.
18. $\frac{1}{12}$.
19. $\frac{5}{13}$.
20. $\frac{49}{120}$.
21. $\frac{1}{2}$.
22. $\frac{2}{5}$.
23. $\frac{1}{4}$.
24. $\frac{3}{10}$.
25. $2\frac{1}{12}$.
26. $\frac{7}{20}$.
27. $\frac{7}{15}$.
28. $\frac{2}{3}$.
29. $2\frac{1}{3}$.
30. $\frac{2}{3}$.
33. $\frac{15}{16}$.
34. $1\frac{1}{3}$.
35. $2\frac{3}{4}$.
36. 33.
37. 35.

Page 111.

17. $\frac{5}{6}$.
18. $\frac{4}{15}$.
19. $\frac{7}{81}$.
20. $8\frac{1}{3}$.
21. $3\frac{1}{7}$.
22. $9\frac{1}{4}$.
23. $8\frac{1}{3}$.
24. $4\frac{1}{6}$.
25. $12\frac{1}{2}$.

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10. $1\frac{1}{3}$.
11. $1\frac{1}{7}$.
12. $\frac{3}{4}$.
13. $1\frac{1}{6}$.
14. $1\frac{1}{5}$.
15. $\frac{2}{5}$.
16. $1\frac{2}{7}$.
17. $\frac{5}{6}$.
18. $3\frac{3}{4}$.
19. $5\frac{5}{9}$.
20. $\frac{6}{25}$.
21. $\frac{5}{6}$.
22. 2.
23. $\frac{24}{35}$.
24. $1\frac{11}{24}$.
25. $\frac{4}{5}$.
26. $1\frac{1}{4}$.
27. 7.
28. $\frac{27}{35}$.
29. $10\frac{2}{3}$.
30. $1\frac{4}{5}$.
31. $9\frac{3}{5}$.
32. $10\frac{1}{2}$.
33. 8.

Page 114.

31. $\frac{75}{4}$.
32. $5\frac{13}{20}$.
33. $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$.
34. $1\frac{2}{3}$.
35. $152\frac{1}{15}$.
36. $17\frac{3}{4}$.
37. $\frac{9}{14}$.
38. 5536.

Page 115.

39. $\frac{7}{10}$.
40. $1\frac{1}{8}$.
41. $1\frac{5}{16}$.
42. 1650 feet.
43. $3750.
44. 5280 feet.
45. 1100 yards.
46. $525.
47. $21000.
48. $3500.
49. 350 bushels.
50. $2400; $1200.
51. $74\frac{17}{24}$ acres.
52. $2\frac{2}{3}$ yards.
53. $\frac{9}{16}$ section.
54. $\frac{19}{80}$ of journey.
55. $\frac{5}{12}$ of farm.
56. $17600.

Page 116.

57. 714 pounds.
58. 48 barrels.
59. 60 days.
60. 100 steps.
61. $1700; $2550.
62. $\frac{1}{6}$ of estate.
63. 3d, 44 pupils.
64. $64.
65. $\frac{5}{16}$.
66. Youngest, 48 A.
67. $62\frac{1}{2}$ miles.
68. 96 pounds; $32.

DEC. FRACTIONS.

Page 120.

37. 45.52
38. 40.045
39. 200.000079
40. .000279
41. 5000.000066
42. .005066
43. 14.00014
44. .001087
45. .0842

Page 121.

2. $\frac{1}{4}$.
3. $\frac{3}{4}$.
4. $\frac{1}{8}$.
5. $\frac{5}{8}$.
6. $\frac{3}{8}$.
7. $\frac{1}{40}$.
8. $\frac{7}{200}$.
9. $\frac{3}{40}$.
10. $\frac{3}{125}$.
11. $\frac{9}{40}$.
12. $\frac{11}{40}$.
13. $\frac{1}{400}$.
14. $\frac{1}{80}$.
15. $\frac{3}{200}$.
16. $\frac{4}{125}$.
17. $\frac{13}{1000}$.
18. $6\frac{1}{4}$.
19. $13\frac{3}{4}$.
20. $4\frac{1}{8}$.
21. $25\frac{3}{8}$.
22. $2\frac{3}{40}$.
23. $216\frac{1}{2}$.
24. $24\frac{7}{20}$.
25. $30\frac{8}{25}$.
30. .5
31. .2
32. .75
33. .6
34. $.6\frac{2}{3}$
35. .625
36. .125
37. .12
38. .075
39. .25
40. .256
41. .06
42. 12.5
43. 56.25
44. 30.75
45. 7.6
46. 6.5
47. 12.25
48. 45.4
49. 24.625

Page 122.

4. 2.626
5. 28.6676

6. 182.771
7. $926.498
8. 306.135 miles.
9. .5378
10. .5786
11. .51694
12. .853735

Page 123.

13. 30.875 pounds.
3. 21.654
4. .0734
5. .058
6. 31.48
7. .174
8. .7992
9. .144
10. .4644
11. .06694
12. 19.125 pounds.

Page 124.

8. .00036
9. .00115
10. .01128
11. .028125
12. 1.8304
13. 1.3824
14. .048375
15. 73.44
16. 2.5331
17. 17.28
18. .4355
19. .18468
20. 161.5
21. 1505.52
22. 1.56
23. 1.3332
24. .000056
25. .03625
26. 123.178
27. .021
28. 3.1828
29. .00156

Page 126.

8. 36.
9. 112.
11. 4.2
12. .26
14. 400.
15. 100.

Page 127.

16. 425.
17. 7000.
18. 23000.
19. 29.1
20. 230.
21. 800.
22. 1.12
23. .36
24. 10.8
25. 750.
26. .84
27. .075
28. .064
29. .12
30. 800.
31. 70.
32. .44

Page 128.

1. .016½, or .0165
2. $\frac{13}{40}$.
3. .32
4. 16.75
5 99.7125
6. 957.882
7. 70470.
8. .000102
9. 40.3
10. 9 hours.
11. 40.4 bushels.
12. .05

U. S. MONEY.

Page 131.

2. $17.82½
3. $143.78½
4. $457.70
6. $.49¾, or $.4975
7. $98.744
8. $9.90

Page 132.

9. $327.75
10. $1.37½
11. $2.50
12. $4.87½
13. $493.25
14. $2.75
15. $881.73
2. $120.
3. $56.
4. $144.90

Page 133.

5. $8.43¾
6. $175.
7. $1200.
8. $106.65
10. $.12, or 12 cts.
11. $37.50
12. $7.50
13. $11.25
15. 60 bushels.

16. 200 lemons.
17. 100 days.
18. 244 bushels.

Page 135.

11. $103.43
12. $600.
13. $40.46875
14. $284.206
15. $277.20
16. 200 yards.
17. 15 pounds.
18. $1.75

Page 136.

19. $288.
20. 56 yards.
21. $.13 per bushel; $1950.
22. $405.
23. $65.
24. $714.
25. $116.95

Page 137.

1. $13.88
2. \$126.94$\frac{1}{2}$

Page 138.

3. \$61.47$\frac{1}{2}$
4. $183.75
5. \$19.12$\frac{1}{2}$

DENOMINATE NUMBERS.

Page 140.

19. $4.48
20. $2.
21. 9$\frac{3}{4}$ bushels.

Page 141.

21. $2.40
22. 112 tumblers.
23. 84 vials.
24. $2.
25. $1.

Page 142.

10. $\frac{1}{2}$ mile.
11. 240 rods.
12. $\frac{5}{3}$ yd., or 1$\frac{2}{3}$ yd.
13. 5 feet.
14. 54 feet.

Page 144.

6. 48 sq. yards.
7. 160 acres.
8. 1600 sq. rods.
9. 3 acres.
10. 60 sq. rods.
11. $\frac{3}{4}$ acre.
2. 42336 cu. inches.
3. 2295 cu. feet.
4. 2601 cu. feet.

Page 145.

5. 9 cu. feet.
6. 13 cu. yards.
7. 684 cu. feet.
8. 12 cords.

Page 146.

11. 930 min.
12. 48 hours.
13. 8760 hours.
14. 31622400 sec.
15. 31557600 sec.
16. 696 hours.
17. 39447000 times.
18. 184 days.
19. 720 hours.
20. 2208 hours.
21. 29220 days.

Page 147.

4. 115 cwt.
5. 14 ounces.
6. 12 cwt.
7. 147 pounds.
8. 10 barrels.

Page 148.

4. 90 years.
5. 3600 pencils.
6. 480 vests.

Page 149.

1. 66000 yards.
2. 23040 acres.
3. 4$\frac{1}{2}$ tons; $78.75
4. $11200.
5. 120 gross.
6. 1584 steps.
7. 3520 times.
8. 12$\frac{17}{44}$ miles, or 12.38 + miles.
9. 240 rods.
10. 2678400 times.
11. 7305 hours.
12. 21 jugs.
13. $1.
14. $11.40
15. $36.60
16. 3500 gallons.

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8. 1500 sq. ft.; 166$\frac{2}{3}$ sq. yards.
9. 14 acres.

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10. 92 sq. yards.
11. $22\frac{1}{2}$ or 22.5 acres.
12. 30 sq. yards.
13. 99 sq. rods.
14. 732 sq. rods;
$4\frac{23}{40}$ acres.
15. 8000 sq. rods;
50 acres.
16. 160 acres.
17. 240 acres.
18. 670 sq. feet;
$74\frac{4}{9}$ sq. yards.

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14. 1872 cu. feet.
15. 1152 cu. feet.
16. 220 cu. yards.
17. $87\frac{1}{2}$ cu. feet.
18. 200 cu. feet.
19. 228 cu. yards;
\$76, cost.
20. 12 cords.
21. $7\frac{1}{2}$ cords;
\$33.75, cost.

PERCENTAGE.

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8. \$3.15
9. 17.6 feet.

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10. 3.3 yards.
11. \$2.25
12. $2.43\frac{1}{3}$ days.
13. .516 feet.
14. 6.118 miles.
15. 2.64 acres.
16. 90 yards.
17. \$2.31
18. \$25.60
19. The number.
20. .066 tons.
21. \$1587.50
22. 147 sheep.
23. 190 acres.
24. 88 girls.
25. 264 pupils.
26. 713 pupils.
27. 1029 bu. wheat;
588 bu. oats;
833 bu. corn.
28. 416 bushels.
29. \$558.
30. \$513.
31. \$11750.

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32. \$175.
33. \$40.
34. \$635.
35. \$93.60
36. \$41.
38. \$18.75
39. \$35.
40. \$12225.
41. \$375.
42. \$519.35
43. \$.99; \$1.10;
\$1.21
44. \$2.88; \$3.12;
\$3.60

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6. \$18.75
7. \$15.39
8. \$22.10
9. \$13.72
10. \$9.054
11. \$150.20
12. \$11.55
13. \$5.76

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14. \$22.14
15. \$40.

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11. \$.014
12. \$.106
13. $\$.129\frac{1}{6}$
14. $\$.198\frac{1}{3}$
15. \$.069
16. $\$.130\frac{1}{3}$
17. $\$.065\frac{1}{6}$
18. $\$.036\frac{1}{6}$
19. $\$.300\frac{5}{6}$
21. \$19.146 +

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22. \$1.701 +
23. \$4,232 +
24. \$24.791 +
25. \$13.50
26. \$87.975
28. \$194.6816 +
29. \$14.546 +
30. \$301.44
31. \$148.50
32. \$303.0625

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